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Analysis of the properties of the topological Index using (analysis tools)

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Keywords: Index, Dendrimers, Core, Stage, PAMAM, Graph.

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Abstract:

Graph G has two sets of information: the vertices, V(G), and the edges, E(G). The definitions for the Connectivity, Geometric Arithmetic, Atomic Bond, and Sum Connectivity Indices of G:

- The Connectivity index: $X(G) = \sum_{u,v\in E(G)} \frac{1}{\sqrt{deg(u).deg(v)}}$
- The Geometric Arithmetic Index: $GA(G) = \sum_{u,v\in E(G)} \frac{2\sqrt{deg(u).deg(v)}}{deg(u)+deg(v)}$
- The Atom bond connectivity index: $ABC(G) = \sum_{u,v\in E(G)} \sqrt{\frac{deg(u)+deg(v)-2}{deg(u).deg(v)}}$
- The Sum connectivity index: $X(G) = \sum_{u,v\in E(G)} 1/(\sqrt{(deg\ (u) + deg\ (v)\ )}\ )$

were deg(u), deg(v) are a degree of vertices. Dendrimers are synthetic, man-made molecules that are composed of monomers organized in a branching structure. in this article, we calculate Connectivity, Geometric Arithmetic, Atom Bond Connectivity, and Sum Connectivity Index for the PAMAM, POPAM, and HACN1J dendrimers.

Keywords: Index, Dendrimers, Core, Stage, PAMAM, Graph.

تحليل خصائص المؤشر الطوبولوجي باستخدام (أدوات التحليل)

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الخلاصة:

الرسم البياني يحتوي على مجموعتين، مجموعة الرؤوس التي يرمز لها بالرمز *V(G)* ومجموعة الحواف *E(G)* . نعرف Connectivity, Geometric Arithmetic, Atomic Bond, and Sum Connectivity Indices والتي تعرف كالتالي:

- $X(G) = \sum_{u,v \in E(G)} \frac{1}{\sqrt{deg(u).deg(v)}}$ $GA(G) = \sum_{u,v \in E(G)} \frac{2\sqrt{deg(u).deg(v)}}{deg(u)+deg(v)}$
- $ABC(G) = \sum_{u,v \in E(G)} \sqrt{\frac{deg(u)+deg(v)-2}{deg(u).deg(v)}}$
- $X(G) = \sum_{u,v \in E(G)} 1/(\sqrt{(deg\ (u) + deg\ (v))}\ )$

عندما deg(u)، deg(v) هما درجة الرؤوس. المتشعبات عبارة عن جزيئات اصطناعية من صنع الإنسان تتكون من مونومرات منظمة في بنية متفرعة. في هذه المقالة سيتم إيجاد صيغ فهارس تبلوجية خاصة Connectivity, Geometric Arithmetic, Atom Bond Connectivity, Sum Connectivity Index لكل من متشعبات ال PAMAM, POPAM وHACN1J. حتى نجد نتائج فهارس هذه التشعبات غير المنتهية التفرع بشكل أسرع وأسهل.

الكلمات المفتاحية: الفهرس ، تشعبات ، المركز ، المرحلة ، *PAMAM* ، الرسم البياني.

1. INTRODUCTION:

A. Balaban (1982) [1], A. Graovac (2010) [2], M. Randi (1975) [3], and N. Trinajsti) (2018) [4] all contributed significantly to the development of chemical graph theory, an essential area of mathematical chemistry. Physical properties, chemical reactions, and biological activities can all be better comprehended with the help of topological indices. From this vantage point, the topological index plays a crucial part in reducing the complexity of the tested molecule to a single real number. Moreover, we graphically present our findings. These visual representations of topological indices highlight the reliance on a specific underlying structure, Graphs can be used to depict chemical substances, A topological descriptor of a graph is a number (or combination of numbers) that quantifies some aspect of the graph. The physicochemical qualities of substances can be investigated with the use of such a descriptor, called a topological index, if it correlates with a certain molecular feature [5]. Recent years have seen extensive research on the analytical and structural features of topological indices in mathematical chemistry. Topological indices are of theoretical and practical significance because they have become a useful tool for investigating a wide range of real-world issues in fields like physics and computer science. In general, graph isomorphism has no effect on topological indices, which are numerical numbers derived from the molecular graph of a chemical compound. There is a class of indices called degree-based topological indices that can be used to highlight and characterize specific aspects of chemical compounds; these indices are calculated by looking at the degrees of the molecular graph. The M-polynomial also has an important role here since it may be used to derive closed-form formulas for degree-based topological indices. The fields of QSPR and QSAR make heavy use of topological indices. Numerous topological indices have been described so far; these indices are crucial to the research of QSPR/QSAR, which in turn aids in the prediction of various physiochemical properties and bioactivity, which is useful in the drug discovery process. It's noteworthy because of all the ways it can be used, including in nano science, biotechnology, and other areas. This receives the attention of researcher's world **[6,7]**. It is helpful to know approximate expressions when topological indices are not feasible. Mathematical characteristics of topological indices have been studied by a Researchers as of late **[8-12]**. The importance of the research from a theoretical point of view is that we find results for these indexes for infinitely branched dendrimers, but from a practical point of view, it is to help medical and analytical researchers in how to use these dendrimers in the field of analysis and pharmacy.

2. The basic concepts:

1- **Definition of the graph:** A simple graph G is vertex-transitive if, for any two vertices of G, there is an automorphism of G that maps one to the other **[13]**.

2- **Definition distance:** the distance between two vertices in a graph is the number of edges in the shortest path connecting them **[14].**

3- Definition degree: The degree of a vertex v in a graph G, denoted deg(v) is the number of proper edges incident on v plus twice the number of self-loops **[15]**.

4- **The definition topological index** is the numerical result of any graph invariant **[16]**.

5- **Definition of dendrimer:** dendrimers represent a unique class of branched polymers, As the generation increases, the external free ends of the previous generation are further branched to produce an exponentially increasing number of new monomers **[17]**.

6- Let G be a graph **the Connectivity index of G** **[13]**, signified by X(G) is:

$X(G)= \sum_{u,v \in E(G)} = \frac{1}{\sqrt{\deg(u).\deg(v)}}$, where deg(u),deg(v) are degree of vertices .

7- Let G be a graph, **the Geometric Arithmetic Index of G [18]**, signified by GA(G) is:

$GA(G)= \sum_{u,v \in E(G)} = \frac{2\sqrt{deg(u).deg(v)}}{deg(u)+deg(v)}$, where deg(u),deg(v) are degree of vertices .

8- Let G be a graph ,**the Atom bond connectivity index of G [19]**, signified by ABC(G) is : $ABC(G)= \sum_{u,v \in E(G)} \sqrt{\frac{deg(u)+deg(v)-2}{deg(u).deg(v)}}$, where deg(u),deg(v) are degree of vertices.

9- Let G be a graph **the Sum connectivity index of G [13]**, signified by X(G) is:

$X(G) = \sum_{u,v \in E(G)} = \frac{1}{\sqrt{deg(u) + deg(v)}}$, where deg(u),deg(v) are degree of vertices

3. Results:

In this section, we will calculate Connectivity, Geometric Arithmetic, Atom Bond Connectivity, Sum Connectivity Index for some of the PAMAM, POPAM, and HACN1J dendrimers. We simply refer to this PAMAM by PD, POMAM by POD2, and HACN 1J by HD1, as will be seen in the shapes that will be presented, and each shape will have several stages of growth of its own. Applications may be mentioned. Farahani MR, Kulli V, Akbari M., Rehman M, Nazeer W, Saleh EA-K, Hameed Jasim T, Raoof AG, Jassim TH in studying some indexes for some manifolds in [8-12]. It is known that a graph can be described by a connection **Table (1)**, a series of numbers, a matrix, a polynomial, or a derived number.

1- we consider type of PAMAM dendrimer, denoted by PD3[3], **Figure (1)** shows that.

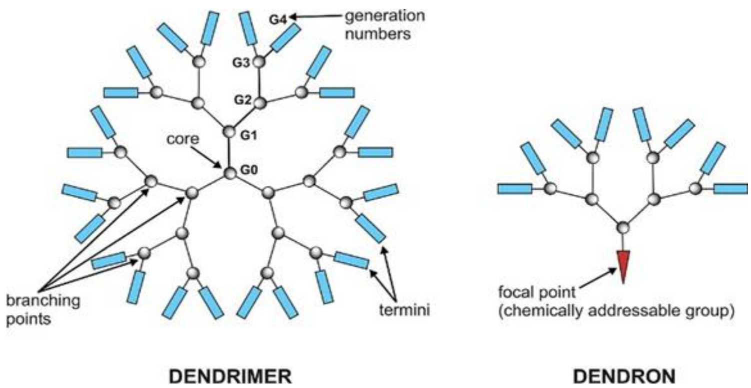

Figure 1. PAMAM dendrimers with 3-developmental stages PD3[3].

Remark 1: |v(PD3[n])|=16×2n+2-467

|E(PD3[n])|= 16×2n+2-468

Table 1: values of (dij) in PD3[n] , (i , j)= (1,3) , (3,3) and steps n= (0,1,2,3)

Stage / di,j	0	1	2	3	n
$d_{1,3}$	3	6	12	24	3×2^n
$d_{3,3}$	0	3	9	21	$3\times2^n-3$

Theorem 1: Let $PD_3[n]$ be PAMAM dendrimers with n developmental stages and n={0,1,2,……..}. Then the Connectivity Index of $PD_3[n]$ is given by $X(G)=(1\backslash\sqrt{3})(3\times2^n)+(1\backslash3)(3\times2^n-3)$.

Proof: for nano star $PD_3[n]$ with contributed (1,3) and (3,3) edge; the formula of Connectivity Index is reduced to: $X(G)=(1\backslash\sqrt{3})(d_{13})+(1\backslash3)(d_{33})$ Using simple calculation; one can show that by remark 1. In each stage, the PAMAM dendrimers PD3[n] edge set can be partitioned into four subsets. Thus, d13 and d33 are the two distinct classes of edges. **Figure (1)** depicts the initial stage graph of PD3[n] (n=0) This is the core of PD3[n]. There are d13 edges and d33 edges totalling 3×2^n and 3×2^n -3. **Table (1)** displays the values of dij for the cases when (I,j)=(1,3),(3,3) and n=0,1,2,3.,there for we obtain $X(G)=(1\backslash\sqrt{3})(3\times2^n)+(1\backslash3)(3\times2^n-3)$.

Theorem 2: Let $PD_3[n]$ be PAMAM dendrimers with n developmental stages and n={0,1,2,...}. Then the Geometric Arithmetic Index of $PD_3[n]$ is given by $GA(G)=(\sqrt{3}\backslash2)[3\times2^n]+[(3\times2^n)-3]$.

Proof: The formula for the Geometric Arithmetic Index of a nano star PD3[n] with a contributed (1,3) and (3,3) edge is reduced to $GA(G)=(\sqrt{3}\backslash2)[d_{13}]+[d_{33}]$ By calculation, can show that by remark 1. In each step, the edge set of PAMAM dendrimers PD3[n] can be partitioned into four partitions. Thus, d13 and d33 are the two distinct classes of edges. The centre of PD3[n] corresponds to the graph of the first stage of PD3[n] (n=0) as depicted in **Figure (1)**. There is 3×2^n and 3×2^n -3 edges, respectively, of types d13 and d33. **Table (1)** displays the values of dij where (I,j) = (1,3),(3,3) and n = 0,1,2,3; hence, we get

$GA(G)=(\sqrt{3}\backslash2)[3\times2^n]+[(3\times2^n)-3]$ □

Theorem 3: Let $PD_3[n]$ be PAMAM dendrimers with n developmental stages and n={0,1,2...}. Then the Atom Bond Connectivity Index of $PD_3[n]$ is given by

$ABC(G)=\sqrt{2\backslash3}(3\times2^n)+(2\backslash3)(3\times2^n-3)$.

Proof: for nano star $PD_3[n]$ with contributed (1,3) and (3,3) edge; the formula for Atom Bond Connectivity Index is reduced to $ABC(G)=\sqrt{2\backslash3}(d_{13})+(2\backslash3)(d_{33})$ Using elementary mathematics, it is possible to demonstrate this using remark 1. The PAMAM dendrimers PD3[n] have four distinct partitions along their edge set. Thus, d13 and d33 are the two distinct classes of edges. The core stage of PD3[n] (n=0) is represented by the graph shown in **Figure (1)**. There are d13 edges and d33 edges totalling 3×2^n and 3×2^n -3. **Table (1)** displays the values of dij for the cases when (I,j)= (1,3),(3,3) and n=0,1,2,3 there for we get $ABC(G)=\sqrt{2\backslash3}(3\times2^n)+(2\backslash3)(3\times2^n-3)$.

Theorem 4: Let PD3[n] be PAMAM dendrimers with n developmental stages and n={0,1,2..}. Then the Sum Connectivity Index of PD3[n] is $X(G)=(1\backslash 2)(3\times 2^n)+(1\backslash\sqrt{6})(3\times 2^n-3)$.

Proof: for nano star $PD_3[n]$ with contributed (1,3) and (3,3) edge; the formula of the Sum Connectivity Index is reduced to $X(G)=(1\backslash 2)(d_{13})+(1\backslash\sqrt{6})(d_{33})$ Using elementary mathematics, it is possible to demonstrate this using remark 1. The PAMAM dendrimers PD3[n] have four distinct partitions along their edge set. Thus, d13 and d33 are the two distinct classes of edges. The initial stage of PD3[n] (n=0) is represented by the graph shown in **Figure (1)**. There are d13 edges and d33 edges totalling 3×2^n -3and3×2^n . **Table (1)** displays the values of dij for the cases when (I,j)=(1,3)(3,3) and n=0,1,2,3 there for we get $H(G)=1\backslash 2(d_{13})+1\backslash 3(d_{33})$.

2-Now, we consider POPAM dendrimers, denoted by $POD_2[n]$. As can be shown in **Figure (2)**, POPAM dendrimers $POD_2[n]$ of generation G_n with three developmental stages.

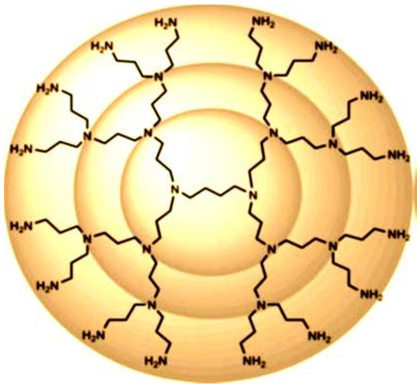

Figure 2: POPAM dendrimer of generations G_n with 3-developmental stages, $POD_2[3]$

Remark 2: $|v(POD_2[n])|=16\times 2^{n+3}-394$

$|E(POD_2[n])|=16\times 2^{n+3}-395$

Table 2: values of (d_{ij}) in $POD_2[n]$, (i , j) = (1,2),(2,2)and(2,3) and steps n = 1,2,3

Stage / di,j	1	2	3	n
$d_{1,2}$	4	8	16	2×2^n
$d_{2,2}$	11	27	59	$8\times 2^n-5$
$d_{2,3}$	6	18	42	$6\times 2^n-6$

Theorem 5: Let $POD_2[n]$ be POPAM dendrimers with n developmental stages and n={0,1,2,…..}. Then the Connectivity index of $POD_2[n]$ is given by $X(G)=(1\backslash\sqrt{2})(2\times 2^n)+(1\backslash 2)(8\times 2^n-5)+(1\backslash\sqrt{6})(6\times 2^n-6)$.

Proof: Connectivity index formula for nano star POD2[n] with (1,2), (2,2), (2,3) edge contributions is $X(G)=(1\backslash\sqrt{2})(d_{23})+(1\backslash 2)(d_{22})+(1\backslash\sqrt{6})(d_{12})$. A quick calculation shows this; see remark 2. In each stage, the POPAM dendrimers' edge set POD2[n] can be partitioned into three subsets. So, we have d12, d22, and d23 edges. The core stage of POD2[n] (n=0) is represented by the graph in **Figure (2)**. The number of edges of types d12, d22, and d23 is 2×2^n, $(8\times2^n)-5$ and $(6\times2^n)-6$. Values of dij are shown in **Table (2)** for the cases where (I,j) = (1,2),(2,2),(2,3), and n = 1,2,3, respectively. then we get $X(G)=(1\backslash\sqrt{2})(2\times2^n)+(1\backslash 2)(8\times2^n-5)+(1\backslash\sqrt{6})(6\times2^n-6)$.

Theorem 6: Let $POD_2[n]$ be POPAM dendrimers with n developmental stages and n={0,1,2...}. Then the Geometric Arithmetic Index of $POD_2[n]$ given to $GA(G) = (2\sqrt{2}\backslash 3)[2\times2^n]+(8\times2^n-5)+(2\sqrt{6}\backslash 5)[(6\times2^n)-6]$.

Proof: for nano star $POD_2[n]$ with contributed (1,2),(2,2) and (2,3) edge ; the formula of Geometric Arithmetic Index is reduced to $GA(G)=(2\sqrt{2}\backslash 3)[d_{23}]+[d_{22}]+(2\sqrt{6}\backslash 5)[d_{12}]$. A quick calculation demonstrates this; see remark 2. In each stage, the POPAM dendrimers' edge set POD2[n] can be partitioned into three subsets. So, we have d12, d22, and d23 edges. The initial stage of POD2[n] (n=0) is represented by the graph in **Figure (2)**. The number of edges of types d12, d22, and d23 is 2×2^n , $(8\times2^n)-5$ and $(6\times2^n)-6$. Values of dij are shown in **Table (2)** for the cases where (I,j)=(1,2),(2,2),(2,3), and n=1,2,3, respectively. then we get $GA(G)=(2\sqrt{2}\backslash 3)[2\times2^n]+(8\times2^n-5)+(2\sqrt{6}\backslash 5)[(6\times2^n)-6]$.

Theorem 7: Let $POD_2[n]$ be POPAM dendrimers with n developmental stages and n={0,1,2,..}. Then the Atom Bond Connectivity Index of $POD_2[n]$ is given by $ABC(G)=\sqrt{1\backslash 2}\,(16\times2^n-11)$.

Proof: Atom Bond Connectivity Index formula reduced for nano star POD2[n] with contributed (1,2), (2,2), and (2,3) edge $ABC(G)=\sqrt{1\backslash 2}\,[d_{12}+d_{22}+d_{23}]$. Using simple mathematics; this is demonstrated by remark 2. The POPAM dendrimers POD2[n] edge set can be partitioned into three parts at each phase. Therefore, there are three varieties of edges: d12, d22, and d23. The core of POD2[n] represents the graph of the initial stage, which is POD2[n] (n=0) as shown in **Figure (2)**. There are2×2^n edges of type d12, $(8\times2^n)-5$ edges of type d22, and $(6\times2^n)-6$ edges of type d23. **Table (2)** displays the values of dij when (I,j) = (1,2),(2,2), and (2,3) and n = 1,2,3; consequently, we obtain $ABC(G)=\sqrt{1\backslash 2}\,(16\times2^n-11)$.

Theorem 8: Let $POD_2[n]$ be POPAM dendrimers with n developmental stages and n={0,1,2...}.

Then the Sum Connectivity index of $POD_2[n]$ is given by $X(G)=(1\backslash\sqrt{3})(2\times2^n)+(1\backslash2)(8\times2^n-5)+(1\backslash\sqrt{5})(6\times2^n-6)$.

Proof: for nano star $POD_2[n]$ with contributed (1,2), (2,2) and (2,3) edge; the formula of Sum Connectivity index is reduced to $X(G)=(1\backslash\sqrt{3})[d_{12}]+(1\backslash2)[d_{22}]+(1\backslash\sqrt{5})[d_{23}]$. Remark 2 demonstrates this via simple calculation. In each stage, the edge set of POPAM dendrimers POD2[n] can be partitioned into three partitions. As a result, we have three sorts of edges: d12, d22, and d23. The core of POD2[n] represents the graph of the first stage, POD2[n] (n=0), as seen in **Figure (2)**. There is 2×2^n, $(8\times2^n)-5$ and $(6\times2^n)-6$ edges of types d12, d22, and d23, respectively. **Table (2)** displays the values of dij where (I,j)= (1,2), (2,2), and (2,3) and n=1,2,3. there for we get $X(G)=(1\backslash\sqrt{3})(2\times2^n)+(1\backslash2)(8\times2^n-5)+(1\backslash\sqrt{5})(6\times2^n-6)$.

3-Lastly, we consider HACN1J dendrimers, denoted by $HD_1[n]$. **Figure (3)** shows that HACN1J dendrimers $HD_1[n]$ of generation G_n with five growth stages.

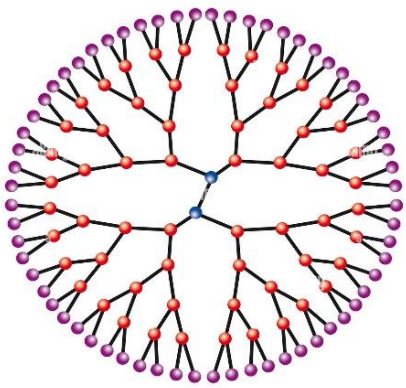

Figure 3: HACN1J dendrimer of generations G_n with 5- developmental stages, $HD_1[5]$

Remark 3: $|v(HD_1[n])|=16\times2^{n+2}-386$

$|E(HD_1[n])|= 16\times2^{n+2}-387$

Table 3: The value of dij in HACN1J where (i , j)=(1, 3)and(3,3) and steps n=1,2,3,4,5

Stage / di,j	1	2	3	4	5	n
$d_{1,3}$	4	8	16	32	64	2×2^n
$d_{3,3}$	1	5	13	29	61	$2\times2^n-3$

Theorem 9: Let $HD_1[n]$ be HACN1J dendrimers with n developmental stages and n={0,1,2….}. Then the Connectivity index of $HD_1[n]$ is given by $X(G)=(1\backslash\sqrt{3})(2\times2^n)+(1\backslash3)(2\times2^n-3)$.

Proof: for nano star HD[n] with contributed (1,3) and (3,3) edge; the formula of Connectivity index is reduced to $X(G)=(1\backslash\sqrt{3})(d_{13})+(1\backslash 3)(d_{33})$. Using easy math; this can be shown in remark 3. In each step, the edge set of HACN1J dendrimers can be split into five parts. So, d13 and d33 are two kinds of edges. **Figure (3)** shows that the core of HACN1J is the curve of the first stage, which is HD1[n] (n=1). There are (2×2^n) and (2×2^n)-3 edges of type d13 and d33. **Figure (3)** shows the values of dij when (I,j)= (1,3) and (3,3) and n = 1,2,3 so we have $X(G)=(1\backslash\sqrt{3})(2\times 2^n)+(1\backslash 3)(2\times 2^n-3)$.

Theorem 10: Let $HD_1[n]$ be HACN1J dendrimers with n developmental stages and n={0,1,2...}. Then the Geometric Arithmetic Index of $HD_1[n]$ is given by $GA(G)=(\sqrt{3}\backslash 2)[2\times 2^n]+(2\times 2^n-3)$.

Proof: fore nanosat HD[n] with contributed (1,3) and (3,3) edge; the formula of the Geometric Arithmetic Index is reduced to $GA(G)=(\sqrt{3}\backslash 2)[d_{13}]+[d_{33}]$. Using elementary calculations; this is demonstrated by remark 3. In each stage, the edge set of HACN1J dendrimers can be divided into five partitions. Therefore, there are two categories of edges: d13 and d33. **Figure (3)** depicts the nucleus of HACN1J, which depicts the graph of the first stage, HD1[n] (n=1). There are (2×2^n) and (2×2^n)-3 edges of d13 and d33, respectively **Figure (3)**, shows the values of dij when (I,j) = (1,3) and (3,3) and n = 1,2,3 then we have $GA(G)=(\sqrt{3}\backslash 2)[2\times 2^n]+(2\times 2^n-3)$.

Theorem 11: Let $HD_1[n]$ be HACN1J dendrimers with n developmental stages and n={0,1,2...}. Then the Atom Bond Connectivity Index of $HD_1[n]$ is given by $ABC(G)=\sqrt{2\backslash 3}(2\times 2^n)+2\sqrt{1\backslash 9}(2\times 2^n-3)$.

Proof: for nano star $HD_1[n]$ with contributed (1,3) and (3,3) edge ; the formula of Sombor index is reduced to $ABC(G)=\sqrt{2\backslash 3}[d13]+2\sqrt{1\backslash 9}[d_{33}]$. Using mathematical consider and by remark3. edges of HACN1J dendrimers can be partitioned into five subsets at each stage. So, there are two distinct kinds of edges: d13 and d33. **Figure (3)** depicts the first stage HD1[n] (n=1) graph near the core of HACN1J(n=1). There are (2×2^n) and (2×2^n)-3 edges of type d13 and d33. **Table (3)** displays the values of dij for the cases where (I,j)=(1,3) and (3,3) and n=1,2 ,3, respectively. there we have $ABC(G)=\sqrt{2\backslash 3}(2\times 2^n)+2\sqrt{1\backslash 9}(2\times 2^n-3)$.

Theorem12: Let $HD_1[n]$ be HACN1J dendrimers with n developmental stages and n={0,1,2...}. Then the Sum Connectivity index of $HD_1[n]$ is given by $X(G)=(1\backslash 2)(2\times 2^n)+(1\backslash\sqrt{6})(2\times 2^n-3)$.

Proof: for nano star $HD_1[n]$ with contributed (1,3) and (3,3) edge; the formula of Sum Connectivity index is reduced to $X(G)=(1\backslash 2)[d_{13}]+(1\backslash\sqrt{6})[d_{33}]$. A quick calculation demonstrates this; see remark 3. In each stage, the HACN1J dendrimer edge set can be partitioned into five subsets. Thus, d13 and d33 are the two distinct classes of edges. As can be seen in **Figure (3)**, the core of HACN1J is represented by the graph of the initial stage, HD1[n] (n=1). The number of edges of types d13 and d33 is (2×2^n) and (2×2^n)-3. When n=1, 2, 3, we get the values of dij shown in **Figure (3)** for (I,j)=(1,3) and (3,3), then we have $X(G)=(1\backslash2)(2\times2^n)+(1\backslash\sqrt{6})(2\times2^n-3)$.

4. Conclusions:

In this research, we have looked at the topological index of several different types of dendrimers, including PAMAM dendrimers, POPAM dendrimers, and HACN1J dendrimers. Topological indexes for various classes of dendrimers are calculated using closed formulas. Our long-term goal is to learn about and calculate topological indices for different classes of dendrimers and nanostructures.

5. Recommendations and future studies :

In this paper, I presented finding new index formulas for new dendrimers, as explained as follows:

1- Finding a new Connectivity index formula special for PD3[n] dendrimer

$X(G) = (1\backslash\sqrt{3})(3 \times 2^n) + (1\backslash3)(3 \times 2^n - 3).$

2- Finding a new Connectivity index formula special for POD2[n]dendrimer

$X(G) = (1\backslash\sqrt{2})(2 \times 2^n) + (1\backslash2)(8 \times 2^n - 5) + (1\backslash\sqrt{6})(6 \times 2^n - 6).$

3- Finding a new Connectivity index formula special for HD1J dendrimer

$X(G) = (1\backslash\sqrt{3})(2 \times 2^n) + (1\backslash3)(2 \times 2^n - 3).$

4- Finding a new Geometric Arithmetic index formula special for PD3[n] dendrimer

$GA(G) = (\sqrt{3}\backslash2)\ [3 \times 2^n] + [\ (3 \times 2^n) - 3].$

5- Finding a new Geometric Arithmetic index formula special for POD2[n]dendrimer

$GA(G) = (2\sqrt{2}\backslash3)\ [2 \times 2^n] + (8 \times 2^n - 5) + (2\sqrt{6}\backslash5)\ [\ (6 \times 2^n) - 6].$

6- Finding a new Geometric Arithmetic index formula special for HD1J dendrimer

$GA(G) = (\sqrt{3}\backslash2)\ [2 \times 2^n] + (2 \times 2^n - 3).$

7- Finding a new Atom bond connectivity index formula special for PD3[n] dendrimer

$ABC(G) = \sqrt{2\backslash3}\ (3 \times 2^n) + (2\backslash3)(3 \times 2^n - 3).$

8- Finding a new Atom bond connectivity index formula special for POD2[n]dendrimer

$ABC(G)=\sqrt{1\backslash 2}\,(16\times 2^n -11)$.

9- Finding a new Atom bond connectivity index formula special for HD1J dendrimer

$ABC(G) = \sqrt{2\backslash 3}[d13] + 2\sqrt{1\backslash 9}[d_{33}]$.

10- Finding a new Sum connectivity index formula special for PD3[n] dendrimer

$S(G) = (1\backslash 2)(3\times 2^n) + (1\backslash\sqrt{6})(3\times 2^n - 3)$.

11- Finding a new Sum connectivity index formula special for POD2[n]dendrimer

$S(G) = (1\backslash\sqrt{3})(2\times 2^n) + (1\backslash 2)(8\times 2^n - 5) + (1\backslash\sqrt{5})(6\times 2^n - 6)$.

12- Finding a new Sum connectivity index formula special for HD1J dendrimer

$S(G) = (1\backslash 2)(2\times 2^n) + (1\backslash\sqrt{6})(2\times 2^n - 3)$.

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