



The Fifth Section, the Semi Parabolic Curves, when the Focus equals the Vertex

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Abstract:

This article introduces a unique case study involving open curves of parabolic form situated within two-dimensional spaces. It presents a new form of a two-dimensional curve achieved by repositioning the focal point to coincide with the vertex position, resulting in what is termed a Semi-Parabolic Curve (SPC) where the focal point acts as the vertex referred to as the SPC head point. In essence, the SPC represents the path traced by a point on a plane, where its distance from a fixed point (the focus), is always greater than or equal to its distance from a fixed straight line (the directrix). Furthermore, the article provides the coordinate equations that govern the points along these curves. With the potential to pave the way for exploring additional geometric aspects relevant to this class of curves, and to enabling comparative analyses across diverse mathematical and geometric domains, particularly in three-dimensional contexts in the future.

Keywords: Circles, Parabolic Curves, Secants, Parabola, Vertex.

القطع الخامس، منحنيات الشبيهة بالقطع المكافئ، عندما تتساوى نقطة
البؤرة مع نقطة الرأس

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الخلاصة:

يدرس هذا البحث حالة جديدة تضم منحنيات مفتوحة في فضاء ثنائي الأبعاد ويقدم شكلاً جديداً من المنحنيات ثنائية الأبعاد متحققاً من إعادة توضيب نقطة التركيز لتتزامن مع موضع الرأس مما يؤدي إلى ما يُعرف بالمنحنى شبه القطع المكافئ (*SPC*) حيث تعمل نقطة التركيز عمل الرأس ويُشار إليها باسم نقطة الرأس. وتأسيساً، يمثل *SPC* المسار الذي ترسمه نقطة على مستوى، حيث تكون مسافتها عن نقطة ثابتة (البؤرة) دائماً أكبر من أو تساوي مسافتها عن خط مستقيم ثابت (المستقيم الموجه).

علاوة على ذلك، فقد قدم البحث المعادلات الإحداثية التي تحكم النقاط على هذا النوع من المنحنيات ووفر القدرة لتمهيد الطريق لاستكشاف جوانب هندسية إضافية ذات صلة بهذا النوع من المنحنيات بما يؤمن مستقبلاً إجراء تحليلات مقارنة عبر مجموعات متنوعة من المجالات الرياضية والهندسية، خصوصاً في سياقات ثلاثية الأبعاد في المستقبل.

الكلمات المفتاحية: دوائر، منحنى القطع المكافئ، خطوط قاطعة، قطع مكافئ، رأس القطعة.

1. Introduction:

The parabola stands out as one of the easiest conic sections to understand, alongside the circle. Initially, Menaechmus used it to replicate the volume of a cube [1-5]. Apollonius, in naming and unraveling numerous crucial attributes, made significant contributions to its study. The parabola earned mentions in the works of Archimedes concerning the calculation of areas under parabolic arcs, found its place in Euclid's writings, and was subjected to scrutiny by Pappus, who delved into the concepts of focus and directrix. Galileo's investigations revealed that projectiles trace graceful parabolic paths through the air [2].

The phenomenon of reflection by parabolic surfaces was analyzed by notable figures such as Gregory and Newton. In modern times, these ideas are employed successfully to find applications in diverse fields, from automotive headlight design to solar cookers, telescopes, astronomical radio dishes, the trajectories of comets, architectural design, and whenever a relationship between two variables involves one being proportional to the square of the other [3]. A parabola is the set of all points in a plane that are equidistant from a fixed line (directrix) and a fixed point (focus) not on the line. The midpoint between the focus and the directrix is called the vertex, and the line passing through the focus and the vertex is called the axis of the parabola. It has a single focus and a directrix, which is a fixed line perpendicular to the axis of symmetry. The distance from any point on the parabola to the focus is equal to its distance to the directrix. Historically, conics have a long history that started with Menaechmus (ca 380- 320 BCE), continued with Euclid and Archimedes, and reached its peak with Apollonius [3-4]. Born about 380 BC in Alopeconnesus, Menaechmus called a parabola a section of a right-angled cone, and hyperbola a section of an obtuse-angle cone [4-5]. In mathematics, a parabola is indicated as a plane curve which is mirror-symmetrical and is approximately U

shaped. On the other hand, classical geometry indeed regards the parabola as a fundamental element within the realm of Euclidean geometry, typically within the framework of real numbers. Numerous treatises and mathematical works have been dedicated to exploring and establishing various properties of the parabola within this mathematical context. Euler made contributions to the study of parabolas and their equations, providing insights into their geometric and algebraic properties [6-7]. Laplace made advancements in applying algebraic methods to the study of conic sections, including parabolas [8-9]. They have significantly contributed to our understanding of this geometric shape and its properties over the real number field, making the parabola a fundamental concept in Euclidean geometry. In general, there are many methods in which the parabola features are constructed. For example, Eccentricity method, rectangular method, Parallelogram method, Tangent method and Offset method [10]. These methods are explained through various examples for drawing a parabola [11]. Nevertheless, geometric constructing methods of parabola are very interesting when using the concentric circles. In parabola, the vertex point, i.e. the parabola head point, is equidistant from a point, the focus, and a line, the directrix. There are several geometric construction methods for drawing a parabola. But here are two commonly used methods: Focus-Directrix Method [11-12], and Vertex-Focus Method [11-13], as geometric construction methods, provide a visual representation of the parabolic curve based on the properties of the focus, directrix, and vertex. By following these steps and constructing multiple points, a parabola can be accurately drawn. Focus-Directrix Method [14] is based on the fundamental definition of a parabola, which states that a parabola is the locus of points equidistant from a fixed point (focus) and a fixed line (directrix). While Vertex-Focus Method focuses on constructing the parabola using the vertex and the focus of the parabola. In Focus-Directrix Method [14-15], the construction process involves drawing the directrix perpendicular to the given line and finding points on the parabola equidistant from the focus and the directrix, whereas Vertex-Focus Method [16] as this method begins with marking the vertex and the focus, followed by constructing points on the parabola equidistant from the vertex and the line connecting the vertex and the midpoint between the focus and the vertex [17-18]. In general, both methods yield accurate representations of the parabola and are widely used in different contexts. The choice between the methods depends on the given information, preferences, and the specific requirements of the problem at hand [19-20].

In this part of the article, a new form of two-dimensional curve is produced by shifting the focus point along the equilibrium axis to be located at the vertex position which means a Semi Parabolic Curve (SPC) whose focus is the vertex point (the SPC's head point). An SPC is the

locus of a point which moves in a plane such that its distance from a fixed point, called the focus, is greater than or equal to the distance from a fixed straight line, called the directrix. An SPC's axis is a straight line passes through the focus and perpendicular to the directrix is called the axis of PSC while the point of intersection of an SPC and its axis is called the vertex of the SPC.

Furthermore, a geometric method in this paper simply flattened the picture and put the series of tangent circles in the same plane as the curve, where one of the segments is held constant. these techniques will be applied to the proposed model of SPC as a special form curve which has a geometric means; hence, its single point simultaneously lies at focus and vertex, and it extends in infinity at both directions of x and y-axis. This paper developed a geometric method and investigated the proportions of this new form of curve, SPC. To draw SPC, let a distance from (F) along the x-axis be a given constant (a), then let the y-axis pass perpendicularly on the symmetrical-axis from the point (F), the origin point (0,0). Let be given a point (F) sets as a vertex point (the PSC head point) at a given constant (AF) on the symmetrical-axis, then on the y-axis construct a line segment (AA₁) with angle (u). Then draw a perpendicular on (AA₁) intersects the x-axis at point (B₁), the midpoint of line segment (AA₁) is point B, which is a point of SPC. These segments are then plotted against the series of geometric means A₁, A₂, A₃, to give a set of the points of B₁, B₂, B₃, all of which lie along y-axis and x-axis respectively. This new curve is named as the SPC.

The vertex is the point of the focus which is the point of intersection of axis and directrix. Besides, if B be a point on the SPC and B_n and BF are the distance from the directrix and focus F point respectively, then the ratio (BF / BA) is called the eccentricity of the SPC, which is denoted by (e). By the parabola definition, the SPC ratio of e is; ($e \geq a$), **Figure 1**.

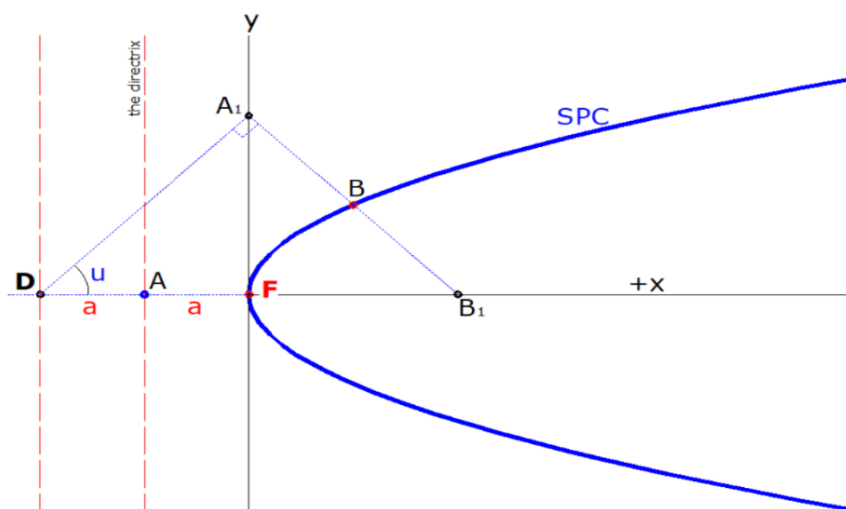


Figure 1. plot the SPC by a set of points in the plane that are equidistant from a SPC's point of B (the midpoint of line segment A₁B₁) which is a perpendicular on a line segment (DA₁) drawn from D with angle (u).

1. SPC's CONSTRUCTION METHODS

A set of geometric methods are produced in this paper to determine points and then to draw the SPC. These three methods are listed as follows:

2.1 Tangents and Circles Centers Method (TCCM):

The TCCM is built according to the proportion of the SPC. The particular principle of TCCM is depending on letting a given distance from the point (A), and the Focus point (the vertex) which is a constant of the SPC, determining the directrix of the SPC on the symmetrical-axis. In addition, the TCCM is applied to the construction of SPC by changing the meanings of tangents and perpendiculars at both side of y and x-axis, then the midpoints of obtained points help to construct the SPC points. The following procedures are presenting the TCCM:

- Let a horizontal distance from (A) to (F) along the x-axis be a given constant, (a), then let the y-axis be the symmetrical-axis from the origin point (F).
- Let be given a point (F) sets as a vertex point, the PSC head point, at a given constant ($AF=a$) on the symmetrical-axis.
- By angle (u), construct line segments from point (D) at a distance of ($2a$), these line segments intersect the y-axis at points (A_n).
- Then draw a perpendicular on (A_nD) from point (A_n), which intersect the x-axis at point (B_n).

By drawing a perpendicular on (A_nD) from point (A_n), a line segment (A_nE_n) is intersecting the y-axis at (A_n) and the x-axis at (E_n). From the midpoint of (A_nB_n), draw a circle that passes through intersection points; (A_n), and (E_n). As a result, the midpoint (B) is a center of the drawing circle, and also it is a point of SPC. Also, the generation method of TCCM benefits to determine as many as possible of points of SPC, and with a help from the French curves the pure curve is precisely drawn, as it is illustrated in **Figure 2**.

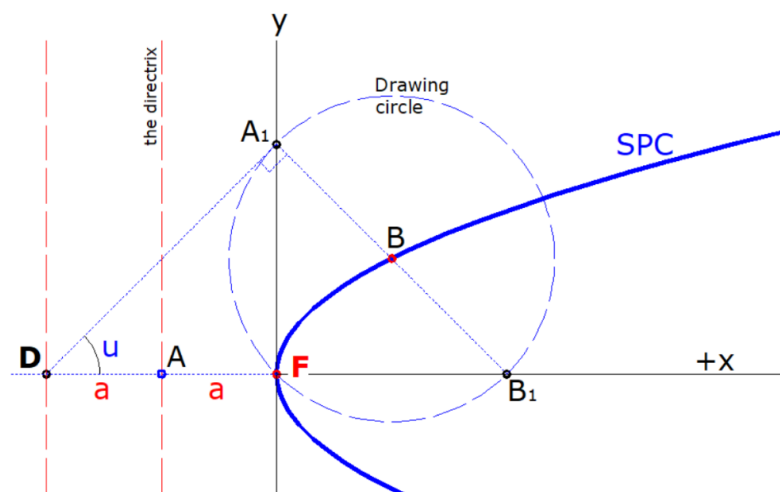


Figure 2.- plot the SPC according to TCCM, where the distance ($AF=a$) is a constant.

TCCM's theorem (1): This theorem states that the SPC's point is the midpoint of the line segment which is drawn from the intersection points with the y-axis and the x-axis, if the line segment is a perpendicular at the y-axis with the line which is drawn from the directrix point on the symmetrical-axis. TCCM generation method is applied to the construction of SPC by the first theorem. To determine the position of a point on the SPC, let (AF) be a given distance where (F) is the origin point. Construct a segment line from (D) to intersect the y-axis at a point (let it be A_1), then from (A_1) draw a perpendicular on (AA_1) which is extended passing through x-axis at (B_1) . Point (B) is the midpoint of the segment (A_1B_1) , which is a point of the SPC, **Figure 2.**

Proof: To prove this theorem, let the segment line (DF) be a given distance on the symmetrical-axis where (F) is the SPC focus which is located on the origin point. Construct a segment line from (D) to intersect the y-axis at point (let it be A_n), then from (A_n) draw a perpendicular on (DA_n) which is the secant passing through coordinate x-axis at (B_n) , then let the point (B) be the midpoint of this segment (A_nB_n) . the triangle (A_nFB_n) is a right-angled triangle. Thus, the circle drawn from the midpoint of the segment (A_nB_n) is passing through three points (F) , (A_n) , and (B_n) ; this means $(A_nB = BB_n)$. Hence, points in the plane that are equidistant from a point B , the midpoint of line segment A_nB_n , is a perpendicular on line (DA_n) with angle (u) . And then, according to the SPC definition, Figure 1, the point of the SPC is the midpoint of the segment (A_nB_n) , **Figure 3.** Hence, points in the plane that are equidistant from a point B , the midpoint of line segment A_nB_n , is a perpendicular on line (DA_n) with angle (u) . And then, according to the SPC definition, Figure 1, the point of the SPC is the midpoint of the segment (A_nB_n) , **Figure 3.**

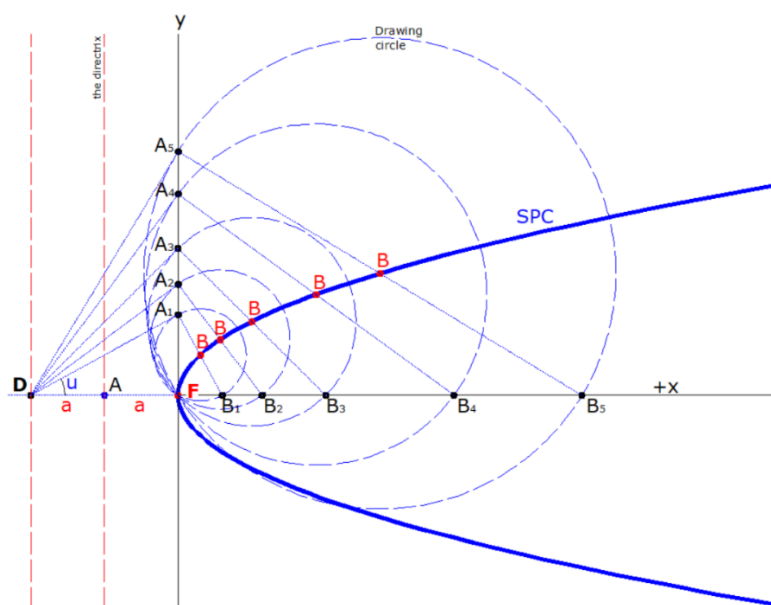


Figure 3.- The plot of the SPC according to TCCM's theorem (1), where; (F) is the focus- vertex point.

It is noticeable that the SPC has a single drawing circle whose center point is the vertex point of the SPC. Therefore, any shared point at the SPC and the parabola segment passes through a single point which is the SPC's vertex and the parabola focus, (F) .

TCCM's theorem (2): states that at any segment whose midpoint is a point of SPC, the line length from the focus to the SPC point or that from the SPC point to either the top or end point of the segment is a constant.

Proof: To proof this theorem, let (AF) be a given distance where (F) is the SPC focus located on the origin point $(0,0)$. Then let (DF) be a segment line of $(2a)$ located on x-axis. From theorem 1, the SPC' drawing circle whose diameter is the segment (A_nB_n) , thus the drawing circle passes through 3 points: (F) , (A_n) and (B_n) . Then, from drawing a circle definition, the diameter (A_nB_n) is divided into two equal parts: $(A_nB = BB_n)$, then from these three points, the $\Delta(A_nB_nF)$ is a right angled triangle at the focus- vertex point (F) , **Figure 4**.

Accordingly, the segments;

$$(A_1F) \perp (FB_n),$$

then from the drawing circle definition;

$$(A_1B = BF = BB_1),$$

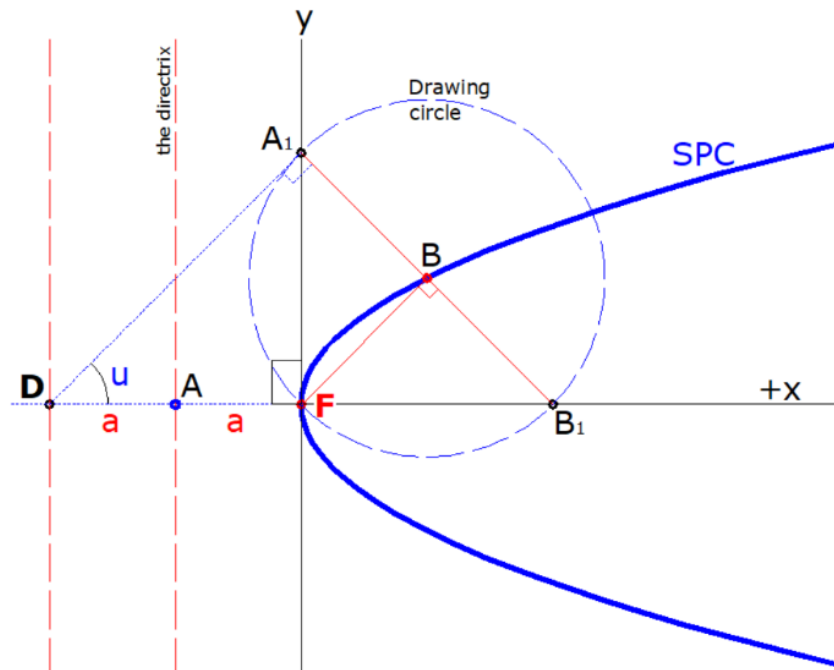


Figure 4.- plot the SPC according to TCCM's theorem (2), where; (F) is the focus- vertex point and Segments $(A_1B= BF= BB_1)$.

TCCM's theorem (3): states that for any SPC, all secants are intersected at a single point lies on the x-axis at a distance of $(2a)$, from the focus- vertex point.

Proof: To proof this theorem, let (AF) be a given distance where (F) is the SPC focus located on the origin point $(0,0)$. Then let (DF) be a segment line of $(2a)$ located on x-axis. From theorems 1 and 2, the right-angled triangles $\Delta(DFA_n)$ are adjacent triangles shared angle (A_nFD) and they are said to be adjacent to each other along a common side (DF) , which is the horizontal distance from the focus- vertex point; thus, $DF=2a$, , **Figure 3**.

It is noticeable from theorem (3) that the SPC is extended into the infinity along the symmetrical-axis where (F) is the SPC focus located on the origin point $(0,0)$. However, it is a succinct way to say that the length of SPC segment is varied according to constance value of (a) , the horizontal distance from the focus- vertex point (F) and (DF) . Furthermore, any point of SPC is a center of a single circle which is passing through the focus point. Building on the theorem one, all circles whose center point is a point of SPC are passing through 3 points (F) , (A_1) and (B_1) , where (B) is the center point and a point of the SPC.

TCCM's theorem (4): states that from any drawing circle, (whose center is a point of SPC), the horizontal line from its center point parallel to the x-axis and intersects the tangent of the drawing circle at a midpoint which from it the perpendicular is drawn from the focus on the line segment that is constructing the focus and the center point of the drawing circle.

Proof: To prove this theorem, let (B) be a point on SPC, and the drawing circle of SPC whose center point is (B) intersects the y and x-axis at (A_n) and (E_n) respectively. From point (B) construct a line segment parallel to x-axis and perpendicular on (DA_n) , hence line segment (BN_n) intersects (DA_n) at a point (N_n) which lies on the directrix. Then, from SPC definition, $(DA = AF = a)$. From the right-angled triangles $\triangle(DFA_n)$, line segments (N_nA) parallel to y-axis, and divides (DF) ; then,

$$(DA = AF = N_n A = a), \text{ thus; } (DN_n = N_n A_n),$$

This theorem indicates that the triangle whose sides are (N_nB) , (BF) , and (FN_n) is a right angled- triangle, hence the line segment (N_nF) is perpendicular on (FB) , **Figure 5**. Similarly, it indicates that the line segment constructed from any point on SPC and the focus is perpendicular on that line segment constructed from the horizontal line from the SPC focus- vertex point, which intersects the directrix, hence; angle $(BFN_n = 90^\circ)$.

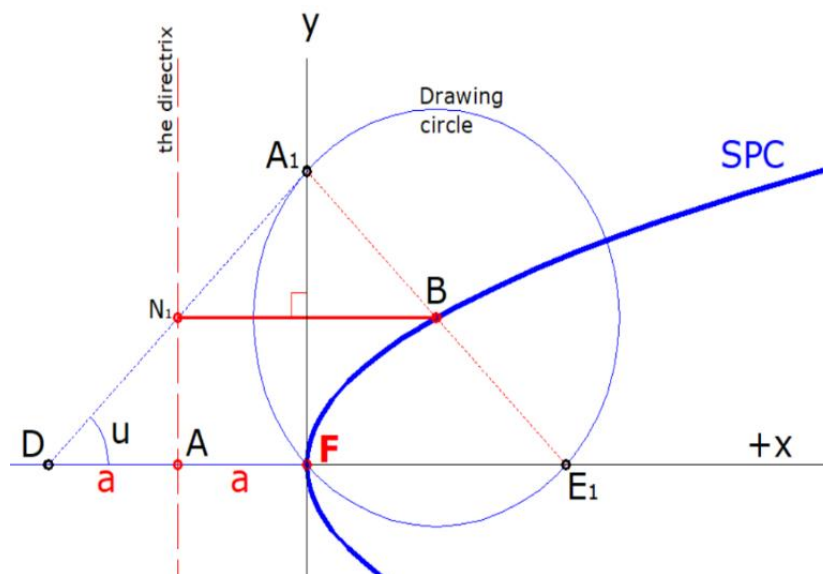


Figure 5.a. - plot the SPC according to TCCM's theorem (4), where;

Where segments; AT and T_nD are tangents on the SPC from points; T and T_n respectively.

TCCM's theorem (6): states that eccentricity of SPC is $e \geq a$:

Proof: To proof this theorem, let (B_n) be a point on SPC, and A and F are the distance from the SPC directrix and focus respectively. From the previous theorem (5), the three points (N_n) , (B_n) and (F) made up a right angled- triangle $\Delta(N_nFB_n)$, where angle $u = \left(\frac{\pi}{2}\right)$. Note that when $(FB_n=0)$, formerly $(e = a)$. Besides, the line segment (FB_n) is the radius of the drawing circle of SPC, then the ratio:

$$e = (r / N_nB_n), \text{ see Figure 7.}$$

Then, the ratio $e = (BF / N_nB_n)$ is called the eccentricity of the SPC, which is denoted by e . By the definition for the SPC, $e \geq a$.

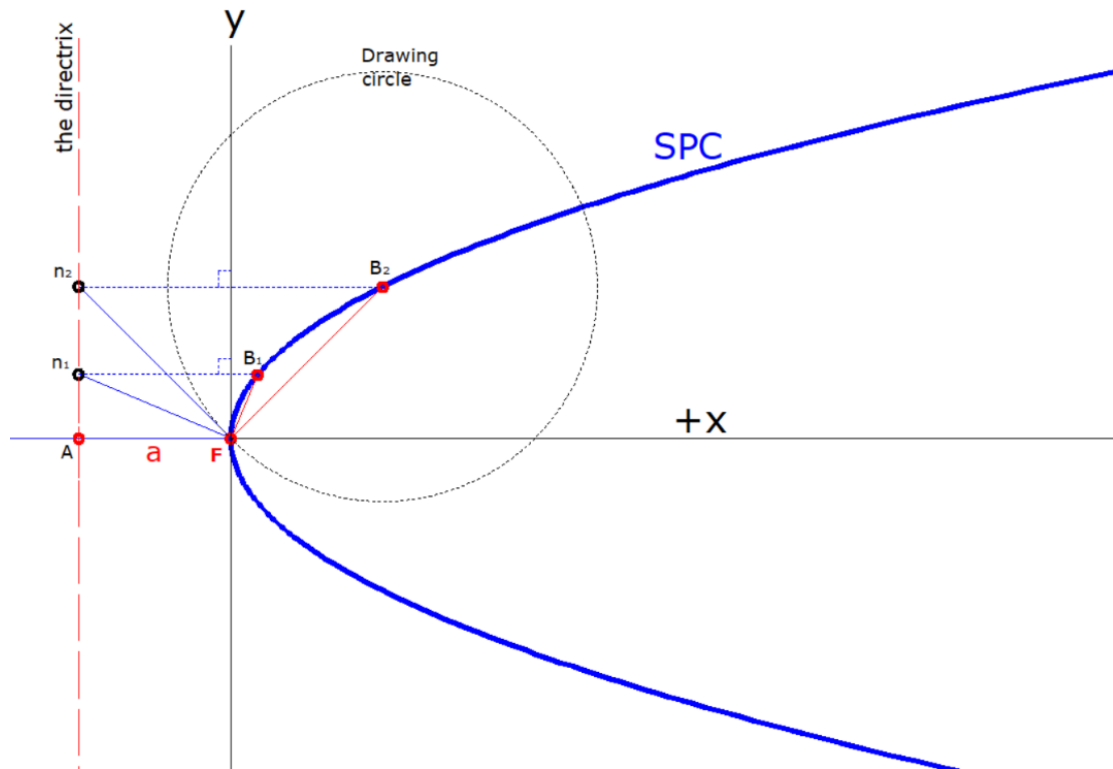


Figure 7. - Eccentricity of the SPC, $e \geq a$

Regarding the parabola segment whose focus is the vertex of SPC, eccentricity of the parabola is $(e = 1)$ which is less than that of SPC. From the origin point $(0,0)$, there is only one tangent which touch SPC from the latus rectum at point $(\pm 2a, \pm a)$, and its length can be determined by the following formula: $\sqrt{3a^2}$, since the SPC drawing circle, whose center is the tangent point (B) has a radius of $(\sqrt{a})$, which lies at $(\pm 2a, \pm a)$, whereas the origin circle of SPC

whose center point is (A), it has radius of (a) and it tangent the vertex-focus point of the SPC, **Figure 7.**

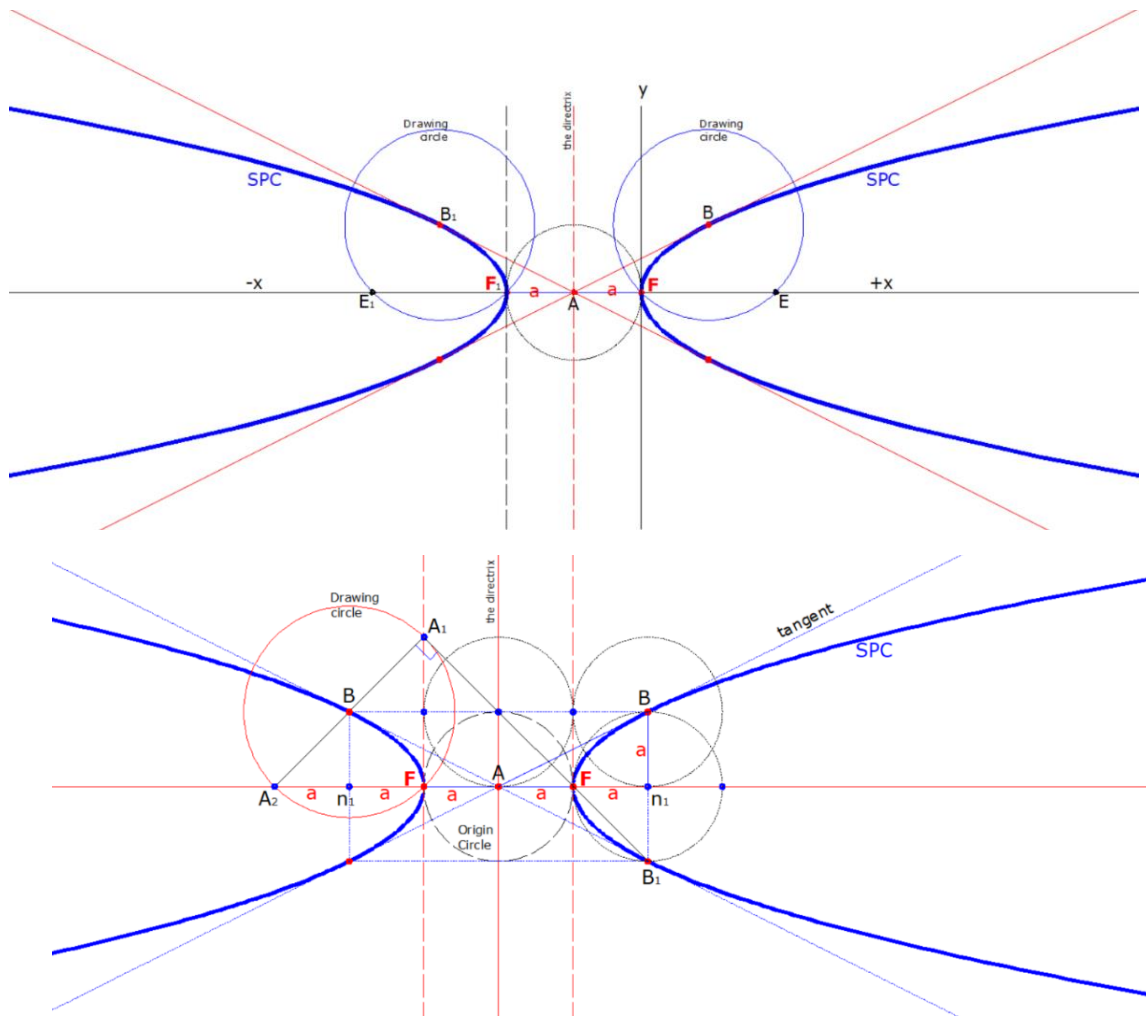


Figure 8. - plot tangents related to SPC latus rectum' point.

Figure 8 illustrates that from the intersection point of the directrix and the x-axis, point (A), is only one tangent to the SPC from the latus rectum' point (B). Hence, the SPC tangent point lies at $(\pm a, \pm 2a)$, since the SPC tangent from the latus rectum' point has a constant slop with a precisely value of ($u = 26.56512^\circ$), where:

$$A_1B = BA_2 = \sqrt{a}, \quad (1)$$

$$A_1B_1 = 3\sqrt{a}, \quad (2)$$

2.2 Tangent Circles & Secant Midpoints (TCSM) construction method:

The TCSM method is built according to the proportion of the SPC. The particular principle of TCSM method is depending on letting a given distance from the point (A), and the Focus point (the vertex) is a constant of the SPC, determining the directrix of the SPC and the vertex point (B) set on the symmetrical-axis. In addition, the TCSM method is applied to the construction of SPC by changing the meanings of tangent circles on the midpoint of their

secants. In this paper, given the focus (F) and the directrix at a constant distance of (a) from point (A), and letting (F) be the origin point of y and x-axis, the TCSM method can construct any number of points on the SPC. The following procedures are presenting the TCSM:

- Let a horizontal distance from (A) to (F) along the x-axis be a given constant, (a), then let the y-axis be the symmetrical-axis from the origin point (F).
- Let be given a point (F) sets as a PSC vertex point, the head point, at a given constant ($DF= 2a$) on the symmetrical-axis, and then from point (D) construct a set of tangent circles at the symmetrical axis starting by the first circle with radius (a), so that the next circles are drawn with increased radius gradually, each circle intersect the y-axis and x-axis at points (A_1) and (E_1) respectively.
- Construct secants from point (A_1) at y-axis to link that point (E_1) at the x-axis.
- From the midpoint of secants (A_nE_1), the midpoint of the secant (B_1) is a point of the SPC, **Figure 9**.

In order to obtain the other points of the SPC across the symmetrical axis, repeat the previous steps at the symmetrical axis to determine these opposite points. Besides, the generation method of TCSM benefits to determine as many as possible points of SPC, and with the help from the French curves, the pure curve is precisely drawn, as it is illustrated in **Figure 9**.

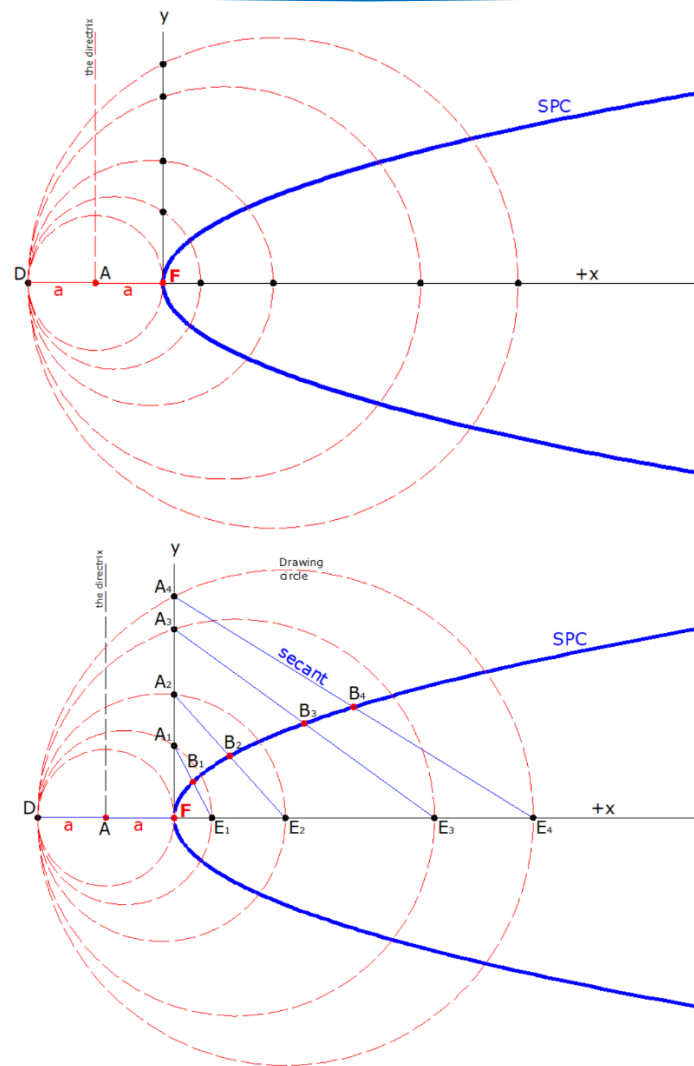


Figure 9. - plot the SPC according to TCSM, where the SPC vertex- focus is (F), and ($DF= 2a$).

It is perceptible that both methods, (TCSM) and (TCCM), can work together to produce SPC points; hence, drawing procedures of both methods are built on a set of circles. However, all points of SPC can be obtained without drawing circles since PSC points lie on the midpoints of secants, (in TCSM), or by a set of perpendiculars midpoints at these segments drawn from y-axis and point (D), (in TCCM), **Figure 10**. Correspondingly, the SPC points can be determined according to two constants: the first constant is (a), and then the distance between (D) and (F), ($2a$). The second constant is angle (u) which determines all secants lengths and then its midpoints which are the SPC points. Additionally, value of (a) determine the case in which the SPC is a concave up curve or a concave down.

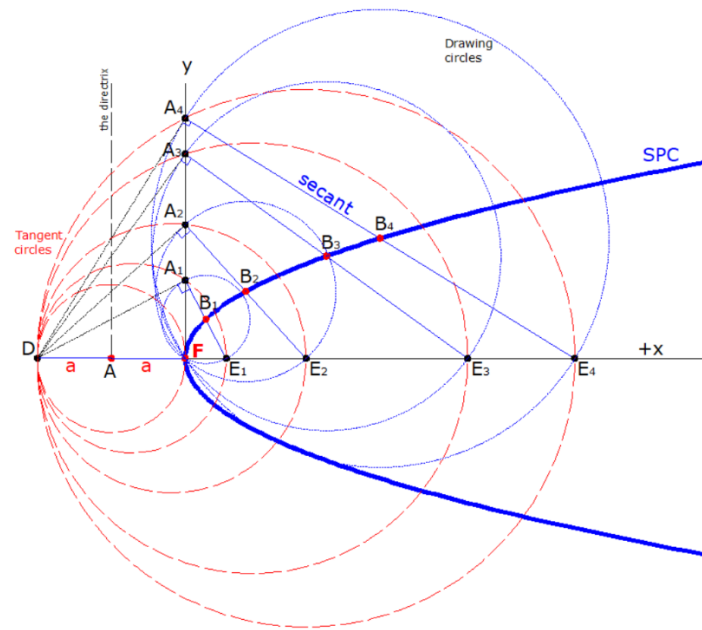


Figure 10. - plot the SPC by both TCSM & TCCM's theorems.

3. Relationships between SPC and Parabola

In general, SPC is a curve whose vertex lies at the focus point of a parabola segment. Furthermore, the parabola directrix lies at $(2a)$; hence, the SPC's vertex point is the parabola focus, (F) . Similar to the parabola, the SPC has two orientations, concave up and concave down. The orientation of an SPC can be found from its equation.

- The SPC is concave up if $(a > 0)$ and concave down if $(a < 0)$.
- The y-intercept is where the SPC meets the y-axis.
- The x-intercepts are where the SPC meets the x-axis.

Also, there will be either;

- two x-intercepts.
- exactly one intercept.
- no intercepts.

A reflection of a geometric figure is a transformation that results in a mirror image of it. For example, the SPC is reflected across the x-axis. Each point (x, y) on the original SPC is reflected to $(x, -y)$ on the reflected parabola. When the parabola directrix lies at (a) then the SPC's vertex point is the parabola focus, (F) . Also, when the parabola segment and the SPC share the same point of the vertex, then the midpoints of all these vertical perpendiculars from any points of parabola and the x-axis are SPC' points. This means that the midpoint (B_n) of a vertical segment (P_nD_n) drawn from any point of parabola segment (P_n) to x-axis is a point of SPC, **Figure 11**.

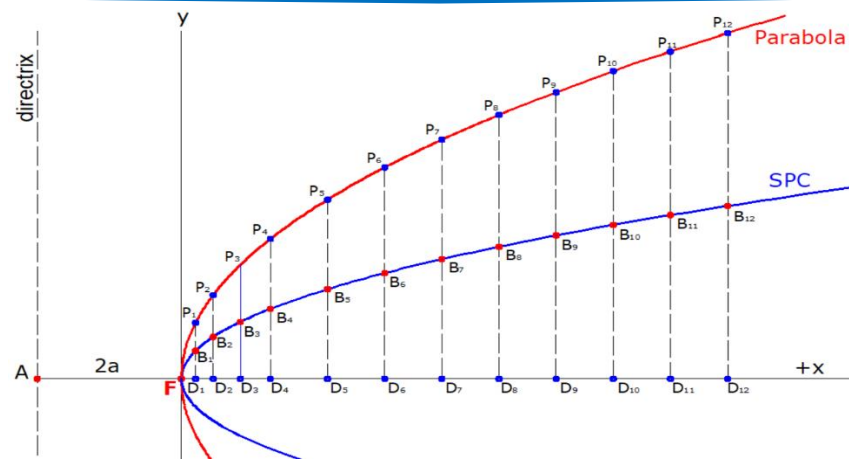


Figure 11. - plot the SPC by a midpoint of perpendicular drawn from any point on parabola to the x-axis, where; $(P_n B_n = B_n D_n)$, if the parabola segment and SPC share the same point of the vertex.

The SPC is a curve whose vertex lies at the focus point in 2-dimensional plane. In general, SPC key proportions are listed in **Table 1**.

Table 1. - Key standard forms of the SPC.

SPC's Standard Equation.	$y = x \tan(90 - u)$, where; $(a > 0)$			
Shape of SPC.				
Vertex point.	$A(0,0)$	$A(0,0)$	$A(0,0)$	$A(0,0)$
Focus point.	$F(0,0)$	$F(0,0)$	$F(0,0)$	$F(0,0)$
Equation of directrix.	$x = -a$	$x = a$	$y = -a$	$y = a$
Equation of axis.	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Length of latus rectum.	$2a$	$2a$	$2a$	$2a$
Extremities of latus rectum.	$(a, \pm a)$	$(-a, \pm a)$	$(\pm a, \pm a)$	$(a, -a)$
Equation of latus rectum.	$x = a$	$x = -a$	$y = a$	$y = -a$
Equation of tangents at vertex.	$x = 0$	$x = 0$	$y = 0$	$y = 0$
Focal distance of $B(x,y)$	$x + a$	$x - a$	$y + a$	$y - a$
Eccentricity (e).	$\geq a$	$\geq a$	$\geq a$	$\geq a$

4. SPC's Standard Equation

By drawing the SPC according to the TCCM, let (a) be the distance of the directrix and the SPC's vertex point (F) along the symmetry axis, where $a \neq 0$, as it is illustrated in **Figure 12**. Then, from the right-angled triangles: (DA_1F) , (A_1FE_1) , and (DA_1E_1) :

$$\cos u = 2a / DA_1, \quad (3)$$

And defined by;

$$DE_1 = 2a + FE_1, \quad (4)$$

Where;

$$\cos u = \left(\frac{\left(\frac{2a}{\cos u} \right)}{2a + FE_1} \right), \quad (5)$$

And we denoted this relation as;

$$2a \cos u + FE_1 \cos u = (2a / \cos u), \quad (6)$$

$$FE_1 \cos u = (2a / \cos u) - 2a \cos u, \quad (7)$$

$$FE_1 \cos u = 2a - 2a \cos^2 u / (\cos u), \quad (8)$$

$$FE_1 \cos u = 2a (1 - \cos^2 u) / (\cos u), \quad (9)$$

Then by equations 1 and 9 we find;

$$FE_1 = 2a (1 - \cos^2 u) / (\cos^2 u), \quad (10)$$

hence SPC' points (x,y) given as follows;

$$x = FE_1 / 2, \quad (11)$$

where; ($a > 0$), then; $\tan (90-u) = (y/x)$,

Then, all points of SPC can be determined by the following formulae:

$$x = \left(\frac{1}{2} \right) \left(\frac{2a (1 - \cos^2 u)}{\cos^2 u} \right), \quad (12)$$

$$y = \left(\left(\frac{1}{2} \right) \left(\frac{2a (1 - \cos^2 u)}{\cos^2 u} \right) \right) \tan(90 - u), \quad (13)$$

The standard equation of SPC's point (x,y) is given as follows:

$$y = x \tan (90 - u), \quad (14)$$

$$x = y / \tan (90 - u), \quad (15)$$

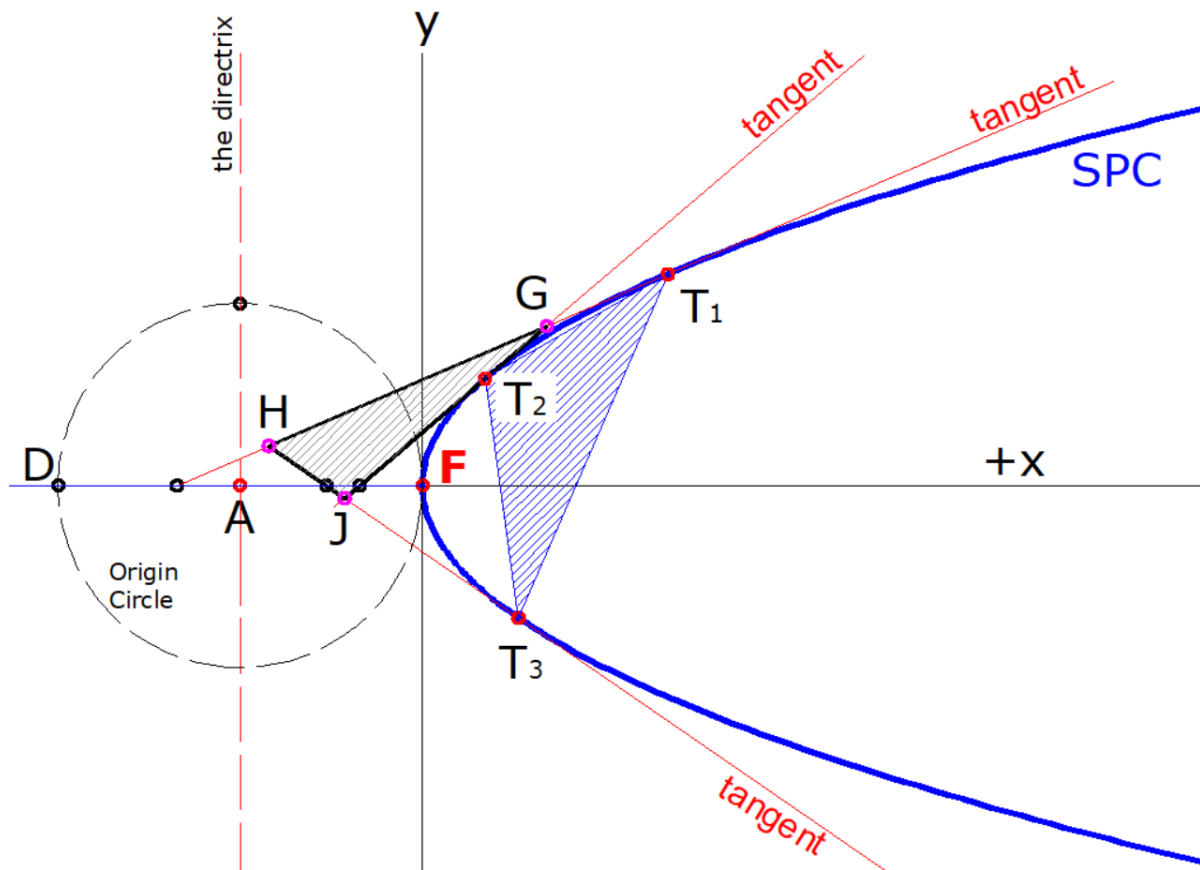


Figure 14: If all three points lie on the SPC, then we have SPC's triangle.

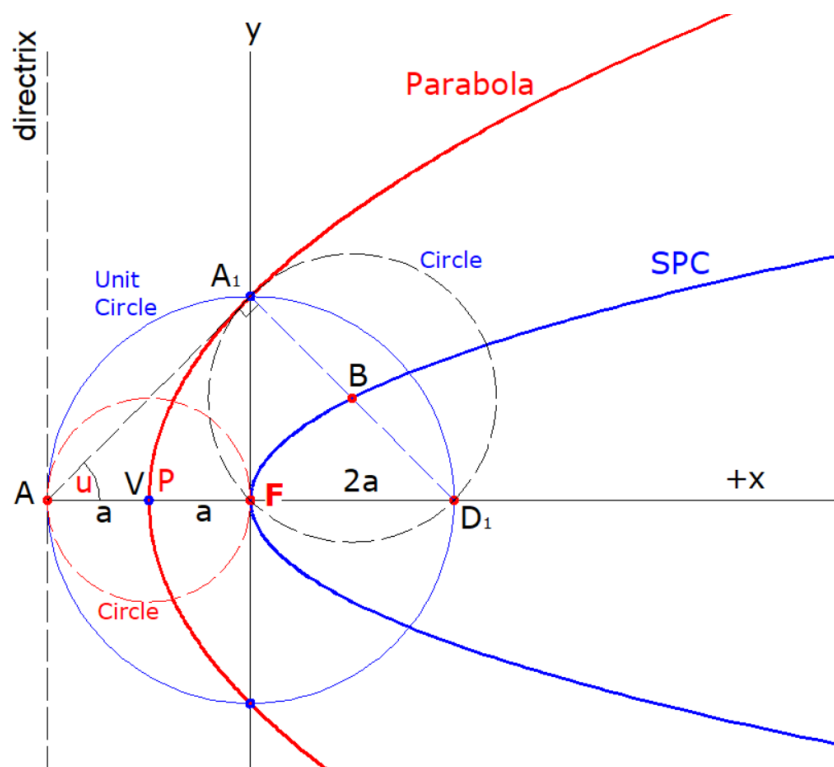


Figure 15 : when both parabola and SPC shared the drawing circle radius, (circle1= circle2).

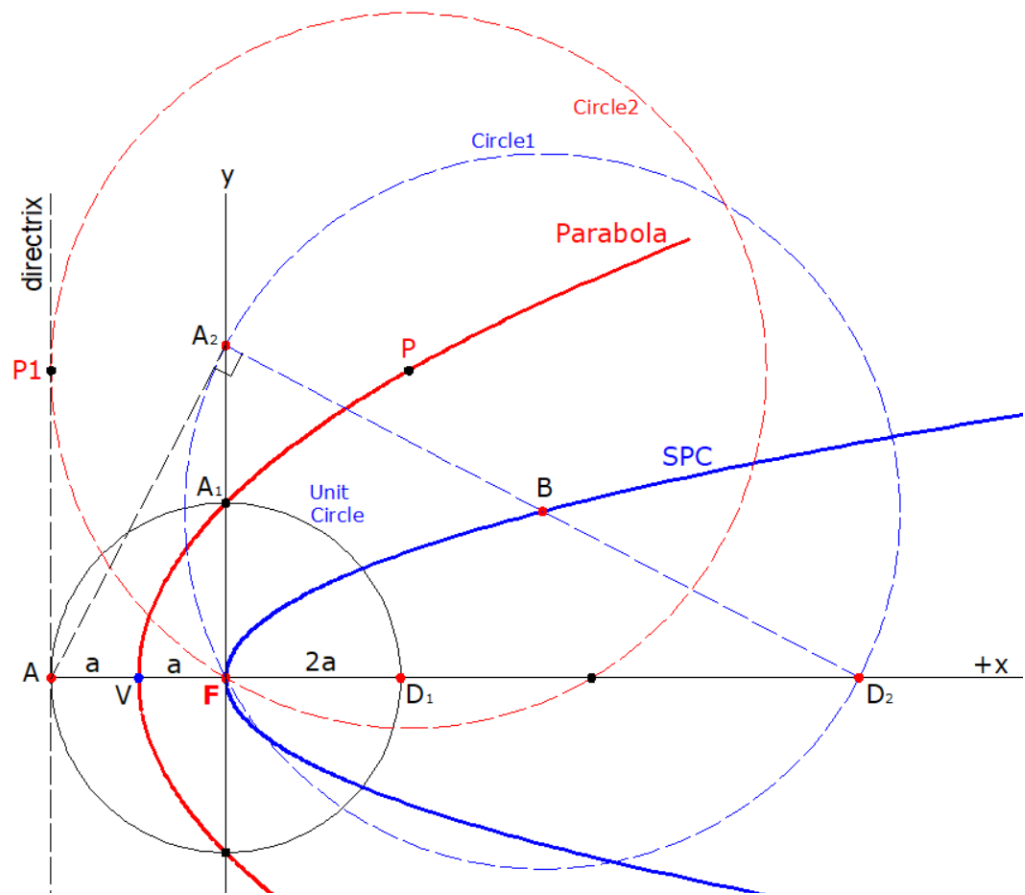


Figure 16: when both parabola and SPC shared the drawing circle radius, (circle1= circle2).

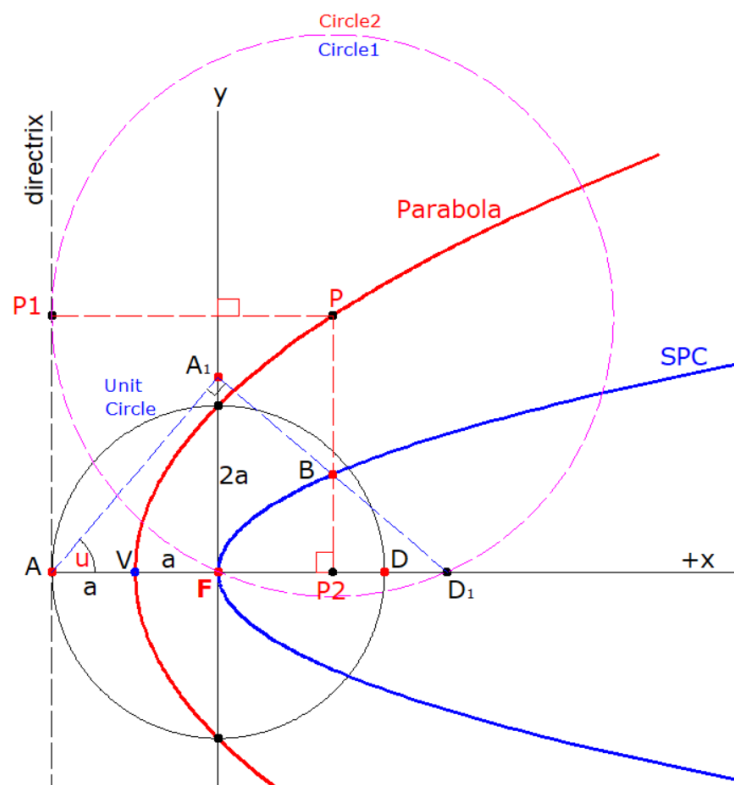


Figure 17 : when the SPC' vertex = parabola's focus

down (if $a < 0$) while from any point at SPC, the drawing circle is intersecting the symmetrical axis at a constant of $(2x)$. Additionally, the radius of any drawing circle equals to $\left(\sqrt{x^2 + y^2}\right)$, correlated to any point of SPC. In general, there is a couple of key integers $[a, u]$ for generating and describing the SPC proportions.

There are several known proofs that have not been introduced in this article, but even though a lot of proofs are known. There is no doubt that there are a lot of undiscovered proofs, and they are worth discussing, sharing and presenting.

Future researches may deeply deal other proportions of this curve, and a list of these proportions are illustrated and listed in **Appendix of special cases of SPC**, for those interested in studying them and applying the result to a generalization of some geometrical theorems related to this form of curve.

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7. References:

- [1] Sanchez WB, Glassmeyer DM. Connecting Parabolas and Quadratic Functions. Math Teach. 2016/2017;110(5):380-386.
- [2] Choi SC. The Universal Parabola. KoG. 2018 Jan; DOI: 10.31896/k.22.4.
- [3] Alkhaldi, Wildberger NJ. The Parabola in Universal Hyperbolic Geometry I. KoG. 2013; pp.14–41.
- [4] Choi SC, Wildberger N. Parabolic Triangles, Poles and Centroid Relations. KoG. 2020; DOI: 10.31896/k.24.1.
- [5] Todhunter I. A treatise on plane co-ordinate geometry as applied to the straight line and the conic section. JO. 2020; Available from: info:edu-jhu-library/collection/brittlebooks/id/barcode/31151007555125.
- [6] Ibragimov NH. CRC Handbook of Lie Group Analysis of Differential Equations Volume I: Symmetries, Exact Solutions, and Conservation Laws. CRC Press; 2018.

- [7] Chindamo R, Ellis J, Peck R. The Mathematics of Parabolas: Focusing on the Vertex Form. *Int J Math Teach Learn*. 2021;22(3).
- [8] Icon DV, Fulier J, Icon O, R Lucia. Note on Archimedes' quadrature of the parabola. *Int J Math Educ Sci*. 2020; Available from: <https://doi.org/10.1080/0020739X.2021.1889700>.
- [9] Al-Ossmi LHM. Elementary Treatise on Melted-like Curves Derived from Center Ellipse and Circle. *Iraqi J Comput Sci Math*. 2023;4(1):114-129. DOI: 10.52866/ijcsm.2023.01.01.009.
- [10] Williams K. Leonhard Euler: life, work and legacy. *Math Intelligencer*. 2018;30:66-69. DOI: 10.1007/BF02985741.
- [11] Wen X. Parabolic higgs bundles on the projective line, quiver varieties and deligne-simpson problem. *Algebraic Geometry (math.AG)*. 2020; Available from: <https://doi.org/10.48550/arXiv.2101.01913>.
- [12] Wen X. Parabolic higgs bundles on the projective line, quiver varieties and deligne-simpson problem. *Algebraic Geometry (math.AG)*. 2021; Available from: <https://doi.org/10.48550/arXiv.2101.01913>.
- [13] Jianping W, Wen X. On the moduli spaces of parabolic symplectic/orthogonal bundles on curves. *Algebraic Geometry (math.AG)*. 2021 Jan 7; Available from: <https://doi.org/10.48550/arXiv.2101.02383>.
- [14] Björn J, La SB, López-M J, L Francesco, Z Myrka. Perceptual judgments of duration of parabolic motions. *Sci Rep*. 2021;11:7108.
- [15] Lyachek Y. An Efficient Method for Forming Parabolic Curves and Surfaces. *Mathematics*. 2020;8(4):592. DOI: 10.3390/math8040592.
- [16] Bialy M. β -Function for ellipses and rigidity. *Entropy*. 2022;24(11):1600. DOI: 10.3390/e24111600.
- [17] Da Silva J, Ramos R. The Lambert-Kaniadakis wk function. *Phys Lett A*. 2020;38(4):126175.
- [18] Haiming L, Jiajing M. Geometric invariants and focal surfaces of spacelike curves in de Sitter space from a caustic viewpoint. *AIMS Math*. 2021;6(4):3177-3204. DOI: 10.3934/math.2021192.
- [19] Sareh M, Gilani S, Gilani N, Abazari Y. Characterizations of dual curves and dual focal curves in dual Lorentzian space $D\{3\}_{\{1\}}$. *Turk J Math*. 2020;44:1561-1577. DOI: 10.3906/mat-1909-6.
- [20] Yücesan GÖ, Tükel J. A new characterization of dual general helices. *ICOM 2020 Conf Proc Book*. 2020; Available from: http://raims.org/files/final_proceedings.pdf.