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Application of Coding Theory in Field 5

Hajir Hayder Abdullah[*1], **Nada Yassen Kasm** [2], **Noor Hussain Abdullah**[1]

[1]Department of Mathematics, College of Education for Pure Science, University of Hamdaniya
[2]Department of Mathematics, College of Education for Pure Science, University of Mosul

Corresponding Author: hajarhayder@uohamdaniya.edu.iq

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Keywords: projective plane, coding theory, incidence matrix, distance, perfect code.

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Abstract:

The main aim of this paper introduce the relationship between the topic of coding theory and the projective plane of order five where special points were found in field 5, which is 31 points, in addition to 31 straight lines, and by applying the theorem that gives the number 1 for the point that lies on the straight line and the number 0 for the point that does not lie on the straight line, we get the code n = 31, d = 6, e = 2, from which we get the table m, v, n. ,n,h, and based on these tables, the distance difference between the code elements was found, where the minimum distance was 6 and the largest distance was 31. These values were used to test the optimality of the code. We can generalize this theorem and apply it to larger fields such as 21 or 23 and others, test their ideality, and find the difference between field 5 and the rest of the fields. M is the maximum value of the size of code over the finite field of order five and an incidence matrix with the parameters, n (length of code), d(minimum distance of code), and e (error-correcting of code)have been constructed some example and theorem have been given.

Keywords: projective plane, coding theory, incidence matrix, distance, perfect code.

تطبيق نظرية الترميز في الحقل٥

هاجر حيدر عبد الله*١، ندى ياسين قاسم٢، نور حسين عبدالله١

١ قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة الحمدانية
٢ قسم الرياضيات ، كلية التربية للعلوم الصرفة ، جامعة الموصل

hajarhayder@uohamdaniya.edu.iq, drnadaqasim3@uomosul.edu.iq, noorhussain@uohamdaniya.edu.iq

الخلاصة:

الهدف الرئيسي لهذا البحث هو تقديم العلاقة بين نظرية الترميز والمستوي الإسقاطي من الرتبة الخامسة حيث تم إيجاد النقاط الخاصة بالحقل ٥ وهي ٣١ نقطة إضافة الى ٣١ مستقيم وبتطبيق المبرهنة التي تعطي الرقم ١ للنقطة الواقعة على المستقيم والرقم ٠ للنقطة التي لا تقع على المستقيم نحصل على الكود $n=31,d=6,e=2$, ومنها نحصل على جدول m ,v ,n ,h, وبالاعتماد على هذه الجداول تم إيجاد فرق المسافة بين عناصر الكود حيث كانت اقل مسافة تساوي ٦ واكبر مسافة ٣١وان هذه القيم تمت الاستفادة منها في اختبار مثالية الكود .ونستطيع تعميم هذه المبرهنة وتطبيقها على حقول اكبر مثل ٢١ أو ٢٣ وغيرها واختبار مثليتها وإيجاد الفرق بين الحقل ٥ وبقية الحقول. M القيمة العظمى لحجم الرمز حول الحقل المنتهي من الرتبة الخامسة . d اقل مسافة للرمز e. تصحيح رمز الأخطاء وأيضا بعض الأمثلة والنظريات.

الكلمات المفتاحية: المستوي الإسقاطي ، نظرية الترميز، مصفوفة الوقوع.

1. Introduction:

This section tackles an explanation of the coding theory in PG(2,5). Coding theory is defined as the study of the properties of symbols and their strengths for specific applications and the polynomial function is also defined as a mathematical structure consisting of one or more constants and variables **[1]**, which are functions whose base algebraic terms or expression While the projective plane is defined as a plan homogeneous space, just as it a plane in space in which plane geometry is achieved, in the research, we have linked them and The theory of coding was presented in the fifth field, which contains the elements (0,1,2,3,4) i.e., they are five in number **[2] [3]**. By multiplying the first point by (1,0,0) by the matrix of its elements from an equation of a certain degree, we obtain the rest of the points that we rely on to obtain the straight lines, and the number of straight lines was 31 **[4] [5]**.

Likewise, by relying on the theorem that gives the value 1 when the point lies on a straight line and gives the value 0 when the point does not belong, thus we obtain the incidence matrix whose elements are 0 and 1. Then we find the distance between each straight line of the projective plane of the fifth order, and from the results, we get different values, where the lowest value is relied upon to test the optimality of the code **[6] [7]**. The results obtained can be relied upon and used in applying a theorem that shows the ideality of the code. These results are considered important because they show us the values of the distance between the code elements and the straight lines. The value of M was also obtained because of its importance in testing the optimality of the code. The motivation for studying the theory of coding was to encode certain elements belonging to a certain field, regardless of the field’s rank in the projective plane, to transform the elements of the field into points that differ in the formula, to find the lines and expressions in the plane, and to test the ideality of those elements if they were converted into a code. The motivation for studying the theory of coding was to encode certain elements belonging to a certain field, regardless of the field’s rank in the projective plane, to transform the elements of the field into points that differ in the formula, to find the lines and expressions in the plane, and to test the ideality of those elements if they were converted into a code.

2. Preliminaries:

2.1 Definition: the plane $\pi = \pi_z$has order q if some line contains exactly z+1 points in the case it follows that π has .[8]

(1 z^2+ z +1 points

(2) z^2+ z +1 lines

(3) z+1 points on a line

(4) z+1 lines through a points

Here is the unique plane =PG(2,z)

2.2 Definition: companion matrix

Let f(x) = x^{n+1}-$a_n x^n$-$a_{n-1}x^n$ …….. a_0 be any manic polynomial then it is companion matrix c(F) is given by (n+1)× (n+1) matrix .[9]

$$C(F) = \begin{bmatrix} 0 & 1 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & . & . & . & . \\ a_0 & a_1 & a_2 & . & . & . & a_n \end{bmatrix}$$

2.3 Theorem: A z- ary (n , M ,2e +1)-code c satisfies.

$$M \{\binom{n}{0} + \binom{n}{1} (z-1) + \dots\dots\dots\dots + \binom{n}{e} (z-1)^e\} \leq z^n. \text{ [10][11]}$$

2.4 Corollary: z- ary (n , M ,2e +1)-code c is perfect if and only if equality holds in theorem 2.3. [12] [13]

3. The classification of cubic over a finite field of order five

The polynomial of degree five $f_2(x) = x^3-2x^2-x+1$ is primitive f_5 ={0,1,2,3,4} since $f_2(0)=1, f_2(1)=4, f_2(2)=4, f_2(3)=2, f_2(4)=4$ and we have by **definition 1** z^2+ z +1 points and z^2+ z +1 lines and z+1 points on a line d q+1 lines through a points t he companion matrix of $f_2(x) = x^3-2x^2-x+1$ in $f_5(x)$ generated the points and lines PG (2,5) as follows. [14] [15]

$$P(k)=[1,0,0]c(g)^{k-1} = [1,0,0]\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

The points of PG(2,5) are:

{ p_1=[1,0,0] , p_2=[0,1,0] , …., p_{30}=[1,2,3] , p_{31}=[1,2,4]} With selecting the point of PG(2,5) which is the third coordinate equal to zero we obtain l_1={1,2,5,7,14,22} and l_kWith selecting the point of PG(2,5) which are the third coordinate[1] equal to zero we obtain l_1={1,2,5,7,14,22} and $l_k = l_1 c(g)^{k-1} = l_1 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$,k= 1,2,…..,31.

Table 1: The line of PG (2,5)

l_1	1	2	5	7	14	22
l_2	2	3	6	8	15	23
l_3	3	4	7	9	16	24
l_4	4	5	8	10	17	25
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
l_{29}	29	30	2	4	12	19
l_{30}	30	31	3	5	13	20
l_{31}	31	1	4	6	14	21

3.1Theorem: the projective plane of order five is a code with a parameter [n = 31 ,d= 6 ,e =2 , m=5^{28}].[16]

***proof*:** the plan $\pi_5$has an incidence matrix A=(aij),where

$a_{ij} = 1$ if $p_i \in l_i$ And also condition $a_{ij} = 0$ if $p_i \notin l_i$

Table 2: Incidence matrix

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	p_{17}	p_{18}	p_{19}	p_{20}	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	p_{26}	p_{27}	p_{28}	p_{29}	p_{30}	p_{31}
l_1	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
l_2	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
l_3	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
l_4	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
l_5	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
l_6	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
l_7	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
l_8	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
l_9	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
l_{10}	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
l_{11}	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
l_{12}	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
l_{13}	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0
l_{14}	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0
l_{15}	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0
l_{16}	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0
l_{17}	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1	0
l_{18}	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	1
l_{19}	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0
l_{20}	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0	0
l_{21}	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	0
l_{22}	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0
l_{23}	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0
l_{24}	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0
l_{25}	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1
l_{26}	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0
l_{27}	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1
l_{28}	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0
l_{29}	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0
l_{30}	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1
l_{31}	1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1

Let,

z={ 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,}

u={1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1}

w={2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2}

f={3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,}

g={4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4} we have

$\mathbf{m_{i=u+l_i}}$

Where

i={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31}

That is:

Table 3: The first element is with the incidence matrix

m_1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1
m_2	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1
m_3	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1
m_4	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1
m_5	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1
m_6	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1
m_7	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1
m_8	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1
m_9	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1
m_{10}	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2
m_{11}	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1
m_{12}	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1
m_{13}	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1
m_{14}	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1
m_{15}	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1
m_{16}	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1
m_{17}	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2	1
m_{18}	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1	2
m_{19}	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1	1
m_{20}	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1	1
m_{21}	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1	1
m_{22}	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1	1
m_{23}	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1	1
m_{24}	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2	1
m_{25}	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1	2
m_{26}	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2	1
m_{27}	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1	2
m_{28}	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1	1
m_{29}	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2	1
m_{30}	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	2
m_{31}	2	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2

W={2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2}

$\mathbf{v_{i=w+l_i}}$

Where

i={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31}

Table 4: The second element is with the incidence matrix

v_{1}	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2
v_{2}	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2
v_{3}	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2
v_{4}	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2
v_{5}	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2
v_{6}	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2
v_{7}	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2
v_{8}	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2
v_{9}	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2
v_{10}	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3
v_{11}	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2
v_{12}	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2
v_{13}	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2
v_{14}	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2
v_{15}	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2
v_{16}	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2
v_{17}	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3	2
v_{18}	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2	3
v_{19}	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2	2
v_{20}	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2	2
v_{21}	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2	2
v_{22}	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2	2
v_{23}	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2	2
v_{24}	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3	2
v_{25}	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2	3
v_{26}	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3	2
v_{27}	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2	3
v_{28}	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2	2
v_{29}	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3	2
v_{30}	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3	3
v_{31}	3	2	2	3	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	3

F={3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3}, $\mathbf{n_{i=f+l_i}}$,Where: i={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31}

Table 5:The third element is with the incidence matrix

n_{1}	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3
n_{2}	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3
n_{3}	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3
n_{4}	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3
n_{5}	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3
n_{6}	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3
n_{7}	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3
n_{8}	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3
n_{9}	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3
n_{10}	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4
n_{11}	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3
n_{12}	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3
n_{13}	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3
n_{14}	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3
n_{15}	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3
n_{16}	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3
n_{17}	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4	3
n_{18}	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3	4
n_{19}	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3	3
n_{20}	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3	3
n_{21}	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3	3
n_{22}	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3	3
n_{23}	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3	3
n_{24}	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4	3
n_{25}	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3	4
n_{26}	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4	3
n_{27}	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3	4
n_{28}	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3	3
n_{29}	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4	3
n_{30}	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4	4
n_{31}	4	3	3	4	3	4	3	3	3	3	3	3	4	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	4

g={4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4}

$\mathbf{h_{i=g+l_i}}$

Where

i={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31}

Table 6: The fourth element is with the incidence matrix

h_1	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4
h_2	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4
h_3	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4
h_4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4
h_5	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4
h_6	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4
h_7	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4
h_8	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4
h_9	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4
h_{10}	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0
h_{11}	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4
h_{12}	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4
h_{13}	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4
h_{14}	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4
h_{15}	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4
h_{16}	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4
h_{17}	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0	4
h_{18}	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4	0
h_{19}	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4	4
h_{20}	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4	4
h_{21}	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4	4
h_{22}	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4	4
h_{23}	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4	4
h_{24}	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0	4
h_{25}	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4	0
h_{26}	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0	4
h_{27}	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4	0
h_{28}	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4	4
h_{29}	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0	4
h_{30}	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0	0
h_{31}	0	4	4	0	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4	4	0	4	4	4	4	4	4	4	4	4	0

Notes:

1- $e = \frac{d-1}{2} = \frac{6-1}{2} = 2$, d=6

2- $M = z^k = z^{31-3}$, k= n-r ,r=3

3- $n = \frac{z^m - 1}{z-1} = \frac{5^3 - 1}{5-1}$,m> 2

4- $n = \frac{z^r - 1}{z-1}$,$k = \frac{q^r - 1}{q-1}$ -r, r=3

4. Results:

By these results **Table 6**, a comparison was made between 31 lines. We notice that the minimum distance is 6, and this value appeared by comparing u with m, z with l, w with v, f with n, and g with h. This means that only these elements can be relied upon in testing the optimality of the code. It must be noted that the largest distance value is 31. This means that the distance value is limited to the range of 6 and 31, d=31 is the highest value.

Table 6: Results

$d(z, l_i) = 6$	$d(g, m_j) = 31$
$d(u, l_j) = 25$	$d(z, v_i) = 31$
$d(w, l_i) = 31$	$d(u, v_i) = 31$
$d(f, l_i) = 31$	$d(w, v_i) = 6$
$d(g, l_i) = 31$	$d(f, v_j) = 25$
$d(l_i, l_j) = 10 \quad i \neq j$	$d(g, v_j) = 31$
$d(z, u) = 31$	$d(w, h_j) = 31$
$d(z, w) = 31$	$d(h_i, l_j) = 31$
$d(z, g) = 31$	$d(m_i, h_j) = 31$
$d(u, w) = 31$	$d(v_i, v_j) = 10 \quad i \neq j$
$d(u, f) = 31$	$d(z, n_j) = 31$
$d(u, g) = 31$	$d(u, n_j) = 31$
$d(w, f) = 31$	$d(w, n_j) = 31$
$d(w, g) = 31$	$d(z, l_j) = 6$
$d(f, g) = 31$	$d(f, n_j) = 6$
$d(u, m_j) = 6$	$d(g, n_j) = 25$
$d(z, m_i) = 31$	$d(n_i, n_j) = 10 \quad i \neq j$
$d(l_i, m_j) = 31$	$d(v_i, m_j) = 31$
$d(m_i, m_j) = 10 \quad i \neq j$	$d(n_i, m_j) = 31$
$d(z, h_j) = 25$	$d(n_i, l_j) = 31$
$d(u, h_j) = 31$	$d(v_i, h_j) = 31$
$d(w, m_j) = 25$	$d(n_i, h_j) = 31$
$d(f, m_j) = 31$	$d(h_i, h_j) = 10 \quad i \neq j$
$d(v_i, l_j) = 31$	$d(v_i, n_j) = 31$
$d(f, h_j) = 31$	$d(g, h_j) = 6$

This value is considered large compared to 6. Therefore, when finding the distance between any two lines, it is preferable to write the elements that make the distance value large in a dedicated table, which we call neglected values or values. That is unnecessary when applying the theorem that depends on the minimum distance.

5. Conclusions:

In the conclusions in **Table 7**, we can notice the difference in the distance values, as the large difference between the field in space and the plane was shown, meaning that the smallest difference is 4 between the fields 3,5,2.

Table 7: The Conclusions

z=5	If we substitute the values of n = 31 ,d= 6 ,e =2 ,in inequality of theorem 1,we get M $=5^{28}$. Hence C is a(31 ,5^{28},6)-code $5^{28}\{\binom{31}{0}+\binom{31}{1}+\binom{31}{2}\}$ $= 5^{28}(1+31(4)+7440)$ $= 5^{28}(7565) < z^n$, n = 31 c is not perfect	
z=3	If we substitute the values of n = 31 ,d= 4 ,e =1 ,in inequality of theorem 1,we get M $=3^{10}$. Hence C is a(13 ,3^{10},4)-code $3^{10}\{\binom{13}{0}+\binom{13}{1}(3-1)\}$ $= 3^{10}(1+26)$ $= 3^{10}$ c is perfect	
Field	Least distance	The different between plane and space
3	4	9
5	6	25
2	3	4

6. References

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