



Using Laplace transforms and their inverses as a graph-based data structure

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Citation: *Ali Farhan Hashoosh¹, Hasan Ramadhan Musa², Omar Osama Dawood³. Using Laplace transforms and their inverses as a graph-based data structure.*

Al-Kitab J. Pure Sci. [Internet]. 2026 Jun. 24 ;10(2):242-254

DOI.

<https://isnra.net/index.php/kjps/article/view/2026>.

<https://doi.org/10.32441/kjps.10.2.p17>

Keywords: Graph theory, Laplace transform, inverse Laplace transform, directed graphs, functional analysis.

Article History

Received 22 May. 2026

Accepted 24 Jun. 2026

Available online 01 Nov. 2026

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Abstract:

In this paper, we present a graph-based structural reformulation of the Laplace transform, referred to as the Laplace Transform Graph (LTG). Within this framework, functions in the time and frequency domains are treated as structured data objects (vertices), while directed edges represent direct and inverse Laplace transform operators.

The LTG provides a unified two-layer mapping structure between functional spaces, explicitly linking each function to its transformed counterpart. The proposed model establishes formal structural properties, including cycle formation, vertex degree characteristics, and convergence constraints induced by transformation consistency.

Computational algorithms are developed to construct and analyze the LTG, enabling path simplification and structural verification. The framework offers a data-oriented perspective on the Laplace transform, bridging functional analysis and graph-based structural modeling for both theoretical interpretation and computational implementation.

Keywords: Graph theory; Laplace transform; inverse Laplace transform; directed graphs; functional analysis.

استخدام تحويلات لابلاس ومعكوساتها كبنية بيانات قائمة على الرسم البياني

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الخلاصة:

في هذا البحث، نقدم إعادة صياغة بنيوية لتحويل لابلاس بالاعتماد على نظرية الرسوم البيانية، أطلقنا عليها اسم **رسم تحويل لابلاس البياني** (Laplace Transform Graph - LTG). ضمن هذا الإطار، تُعامل الدوال في مجالي الزمن والتردد بوصفها كائنات بيانات منظمة (رؤوسًا)، في حين تمثل الحواف الموجهة عمليات تحويل لابلاس المباشر والعكسي.

يوفر نموذج LTG بنية ربط موحدة ثنائية الطبقات بين الفضاءات الدالية، بحيث يربط بصورة صريحة كل دالة بنظيرتها المحوّل. كما يحدد النموذج المقترح خصائص بنيوية رسمية، تتضمن تكوين الدورات، وخصائص درجات الرؤوس، وقيود التقارب الناتجة عن اتساق التحويلات.

وقد تم تطوير خوارزميات حاسوبية لبناء وتحليل رسم تحويل لابلاس البياني، بما يتيح تبسيط المسارات والتحقق البنيوي. ويوفر هذا الإطار منظورًا قائمًا على البيانات لتحويل لابلاس، جامعًا بين التحليل الدالي والنمذجة البنيوية المعتمدة على الرسوم البيانية، بما يخدم التفسير النظري والتطبيقات الحاسوبية على حد سواء.

الكلمات المفتاحية: نظرية الرسوم البيانية؛ تحويل لابلاس؛ تحويل لابلاس العكسي؛ الرسوم البيانية الموجهة؛ التحليل الدالي.

1. Introduction

The Laplace transform is one of the most powerful mathematical tools for solving linear differential equations [1, 2]. In general, a transformation of a function $f(t)$ can be expressed as equation (1):

$$F(s) = \int_a^b K(s, x) f(x) dx \quad (1)$$

where $F(s)$ is the transformation of the function $f(t)$, and $K(s, t)$ is called the transformation kernel[1]. This approach converts the original problem involving $f(t)$ into another problem involving $F(s)$, which is often simpler to solve. By selecting an appropriate kernel and limits of integration, differential equations can essentially be reduced to algebraic equations[3]. A special and widely used case of this idea is the Laplace transform, defined as equation(2):

$$L\{f(t)\} = F(s) = \int_0^{\infty} e^{-sx} f(t)dx \quad (2)$$

where $s = \sigma + j\omega$ is a complex variable, $f(t)$ is the original function in the time domain, and $F(s)$ is its Laplace transform. This method is extensively applied in solving non-homogeneous differential equations, particularly when the homogeneous part is unstable. Beyond its analytical formulation, mathematical transformations can also be interpreted structurally, which motivates the introduction of graph-theoretic representations. On the other hand, graph theory is a fundamental and important field in applied mathematics, computer science, and other related disciplines, and has numerous and wide-ranging applications in algorithm design, knowledge representation, and network analysis. A graph is defined as a non-empty set of vertices V and a non-empty set E of unordered pairs of vertices called edges. Depending on its structure, a graph may be simple, multigraph, loop-like, or contain isolated vertices. A tree is a special type of connected, non-cyclic graph, while a regular r -graph is a graph with the degree of each vertex r [4].

This paper introduces a new perspective by treating mathematical functions as vertices, with the direct and inverse Laplace transforms as edges connecting them. Within this framework, different classes of functions such as linear, logarithmic, power, trigonometric, and inverse trigonometric functions can be structurally connected to form new types of function graphs. The central hypothesis of this work is that the Laplace transform induces a structured mapping between functional spaces that can be formally represented as a directed graph. This novel framework, which we call the Laplace Transform Graph (LTG), establishes a new type of graph structure that integrates concepts from both functional analysis and graph theory [5,6]. Although the Laplace transform has been extensively studied in mathematics, control theory, and signal processing, existing studies primarily focus on analytical, numerical, or application-oriented aspects of the transform. To the best of our knowledge, no previous study has introduced a unified graph-theoretic framework in which mathematical functions are represented as vertices and Laplace transform operations are represented as directed edges connecting time-domain and frequency-domain functions. This gap motivates the development of the Laplace Transform Graph (LTG), which provides a structural representation of transform relationships within a graph-based environment. Although the Laplace transform has been extensively studied in mathematics, control theory, and signal processing, existing studies have primarily focused on its analytical, numerical, and application-oriented aspects. To the best of our knowledge, no previous study has introduced a unified graph-theoretic framework in which mathematical functions are represented as vertices and Laplace transform operations are represented as directed edges connecting time-domain and frequency-domain functions. This

gap motivates the development of the Laplace Transform Graph (LTG), which provides a structural representation of transform relationships within a graph-based environment. The main contribution of this work lies in representing time-domain and frequency-domain functions as graph vertices and modeling direct and inverse Laplace transform operations as directed edges. Furthermore, the proposed LTG integrates both domains within a single graph structure and establishes fundamental structural properties, including cycle formation, vertex degree characterization, connectivity, and component decomposition, thereby providing a new graph-theoretic perspective for analyzing transform relationships. To further clarify the novelty of the proposed framework, a comparison between previous studies and the proposed Laplace Transform Graph (LTG) is presented in Table 1. The comparison highlights the main structural differences and demonstrates the unique features introduced in this work.

Table 1. Comparison between previous studies and the proposed Laplace Transform Graph (LTG).

Feature	Previous Studies	Proposed LTG
Use of Laplace Transform	Yes	Yes
Graph-Based Analysis	Partial	Yes
Functions Represented as Vertices	No	Yes
Laplace Transform Operators Represented as Edges	No	Yes
Unified Time- and Frequency-Domain Representation	No	Yes
Structural Graph Analysis	Limited	Yes

2. Related Works

The use of the Laplace transform is traditional in system analysis and signal processing; however, its representation through graph structures has been addressed only partially in the existing literature. For instance, some researchers proposed graph-based algorithms to compute distributions of coalescence times after Laplace transformation in the context of population genetics, where the graph represents the state space and the Laplace transform is used to derive probability distributions. However, in this context, the vertices do not represent functions themselves, and the edges are not explicitly modeled as Laplace transform operators, which distinguishes our work from these previous studies.

Another study generalized the Laplace transform to signals defined on irregular domains, including graphs and trees, introducing the so-called spectral Laplace transform, which extends the classical Fourier-Laplace analysis to graph-defined data. Nevertheless, in this framework, vertices represent spatial or structural points rather than functional entities in the time or frequency domain, which is fundamentally different from the scope of our research. To date, no study has proposed an explicit graphical framework, termed the Laplace Transform Graph (LTG), in which vertices represent functions in the time or Laplace domain, and edges correspond to direct or inverse Laplace transforms. This work provides a new approach that unifies time-domain and frequency-domain analysis within an integrated graph model, distinguished from previous works by explicitly representing vertices, functions, and edges in a clear and systematic manner. Additionally, previous works by the authors in control theory provide complementary insights into system behavior and stability, which can be related to transformations and graph-based representations of functions:

Routh stability criterion and Lyapunov-Routh method in control theory [2023]

Stability on optimal control problems and multi-population mathematical models [2023]

Special methods for controllability and observability in optimal control systems [2024]

These studies highlight the authors' prior experience in analyzing mathematical systems and provide a foundation for extending graph-based frameworks to functional transformations [7-8-9].

2.1 Laplace Transform Graph

We assume that the vertices of the graph represent mathematical functions, while the edges represent Laplace transform operations applied to these functions. When the Laplace transform is applied to a function $f(t)$ at a given vertex, it produces a new function $F(s)$ that represents another vertex in the graph, and the two functions are connected by an edge representing the transformation(3) [1]:

$$F(s) \xrightarrow{L} f(t) \quad (3)$$

Similarly, the inverse Laplace transform can be represented as an edge in the opposite direction (4):

$$f(t) \xrightarrow{L^{-1}} F(s) \quad (4)$$

In this unified graph model, vertices provide information about functions in the time or frequency domain, and edges represent the mathematical operations connecting them. This structure allows the study of function connectivity, different transformation paths, and potential cycles that may impose constraints on initial values or convergence regions.

Formally, each vertex v_i can be described as the union of its time-domain and frequency-domain representations (5):

$$v_i = (f_i(t), F_i(s)) \quad (5)$$

If functions are described as vertices, then the set of vertices is:

$$V = (v_1, v_2, v_3, \dots, v_n)$$

The edges represent relationships between functions and their Laplace transforms:

$$E \subseteq V \times V$$

according to the applied transform operator., for example:

$$v_1 = f_1, v_2 = f_2, v_3 = f_3, v_4 = f_4, v_5 = f_5$$

2.2 Graph of Direct Laplace Transform

In this section, we explain the direct (forward) Laplace transform in detail and illustrate how functions can be represented graphically using this transform. The general principle is as follows: each vertex represents a function in the time domain $f(t)$, and each edge represents the application of the Laplace transform that maps the function into the frequency domain $F(s)$ [4]. This representation allows us to track the transition of functions into the frequency domain and to visualize their relationships in a graphical form. Moreover, if a graph with its vertices and edges is given, one can infer the corresponding functions in both domains[10-11].

Formally, for each time-domain function $f_i(t)$ represented by a vertex v_i , the Laplace transform produces a new vertex $\hat{v}_i$ representing $F_i(s)$ (6) [5, 6]:

$$\hat{v}_i = F_i(s) \xrightarrow{L} v_i = f_i(t) \quad (6)$$

We define the graph of the direct transform as (7):

$$G_L = (V_L, E) \quad (7)$$

where $V_L = V_t \cup V_s$ is the union of the time-domain vertices $V_t = \{f_i(t)\}$ and the frequency-domain vertices $V_s = \{F_i(s)\}$. The set of edges is defined by the rule (8):

$$(f_i(t), F_i(s)) \in E_L \Leftrightarrow F_i(s) = L\{f_i(t)\} \quad (8)$$

Example1: Given $V_t = \{1, t, e^t\}$ (three time-domain vertices) and total 6 vertices (frequency + time), find V_s and the graph of the direct transform.

Solution: Compute the Laplace transforms:

$$L[1] = \frac{1}{s} = \hat{V}_1, L[t] = \frac{1}{s^2} = \hat{V}_2, L[e^t] = \frac{1}{s-1} = \hat{V}_3$$

So the frequency-domain vertex set is

$$V_s = \left\{\frac{1}{s}, \frac{1}{s^2}, \frac{1}{s-1}\right\}$$

The full vertex set of the direct-transform graph is the union

And the graph of the direct transform

$$V_L = V_t \cup V_s = \left\{1, t, e^t, \frac{1}{s}, \frac{1}{s^2}, \frac{1}{s-1}\right\}$$

hence 6 vertices in total.

The edge set E_L (direct Laplace edges) is given by the transform relations:

$$E_L = \left\{\left(1, \frac{1}{s}\right), \left(t, \frac{1}{s^2}\right), \left(e^t, \frac{1}{s-1}\right)\right\}$$

(We write edges as ordered pairs from time-vertex to frequency-vertex because the direct transform goes forward.)

Graph properties / counts

Number of vertices: $|V_L| = 6$.

Number of edges: $|E_L| = 3$.

The graph is disconnected with 3 connected components (each component is a pair $(f_i(t), F_i(s))$). Edge Set (directed) figure 1:

Disconnected Graph of Direct Laplace Transform (6 vertices, 3 edges)

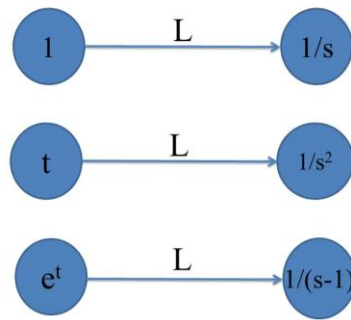


Figure 1. Disconnected graph direct Laplace transform.

The compact formal definition (9):

$$G_L = (V_L, E),$$

$$V_L = V_t \cup V_s, (f_i(t), F_i(s)) \in E_L \Leftrightarrow F_i(s) = L\{f_i(t)\} \quad (9)$$

This example illustrates a type of disconnected graph, since the vertices are grouped in pairs without edges connecting the different groups.

2.3 Inverse Laplace Transform

The inverse Laplace transform is a fundamental concept in mathematical analysis and system theory, as it allows the recovery of a time-domain function $f(t)$ from its frequency-domain representation $F(s)$. In this section, we illustrate how to represent the graph of the inverse Laplace transform, where functions in the time and frequency domains are considered as vertices, and the edges correspond to the inverse transformation operation. The general concept can be expressed as follows(10):

$$v_i = f(t) \xrightarrow{L^{-1}} \hat{v}_i = F(s) \quad (10)$$

This graphical representation provides a visual means of understanding the relationship between the original time-domain function and its frequency-domain counterpart, and it enables the tracking of inverse transformation paths among different functions within the unified graph framework[12].

We define the inverse Laplace transform as (11):

$$G_{L^{-1}} = (V_{L^{-1}}, E_{L^{-1}}) \quad (11)$$

where $V_{L^{-1}} = V_{L^{-1}} \cup E_{L^{-1}}$ is the union of the time-domain vertices $V_t = \{f_i(t)\}$ and the frequency-domain vertices $V_s = \{F_i(s)\}$. The set of edges is defined by the rule:

$$(F_i(s), f_i(t)) \in E_{L^{-1}} \Leftrightarrow f_i(t) = L^{-1}\{f_i(s)\} \quad (12)$$

Example2: Given $V_s = \{\frac{2}{s^2+4}, \frac{3}{(s+3)^2+9}, \frac{1}{s^2}\}$ (three frequency -domain vertices, $(s > 0)$ and total 6 vertices (time + frequency), find V_t and the inverse Laplace transform.

Solution: Compute the inverse Laplace transform:

$$L^{-1}\left[\frac{2}{s^2+4}\right] = \sin 2t = \hat{V}_1, L^{-1}\left[\frac{3}{(s+3)^2+9}\right] = e^{-3t}\sin 3t = \hat{V}_2, L^{-1}\left[\frac{1}{s^2}\right] = t = \hat{V}_3$$

So the frequency-domain vertex set is

$$V_t = \{\sin 2t, e^{-3t}\sin 3t, t, \}$$

The full vertex set of the inverse Laplace transform is the union(12)

$$V_L = V_s \cup V_t = \{\sin 2t, e^{-3t}\sin 3t, t, \frac{2}{s^2+4}, \frac{3}{(s+3)^2+9}, \frac{1}{s^2}\} \quad (12)$$

hence 6 vertices in total.

The edge set E_L (direct Laplace edges) is given by the transform relations (13)

$$E_L = \left\{\left(\frac{2}{s^2+4}, \sin 2t\right), \left(\frac{3}{(s+3)^2+9}, e^{-3t}\sin 3t\right), \left(\frac{1}{s^2}, t\right)\right\} \quad (13)$$

(We write edges as ordered pairs from time-vertex to frequency-vertex because the inverse Laplace transform goes forward.)

Graph properties / counts

Number of vertices: $|V_{L^{-1}}| = 6$.

Number of edges: $|E_{L^{-1}}| = 3$. The graph is disconnected and consists of three weakly connected components (each component is a pair $(F_i(s), f_i(t))$). Since the graph is directed, connectivity is considered in the weak sense.

Graph of the inverse Laplace transform is shown in Figure 2:

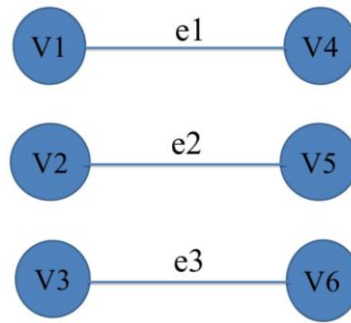


Figure 2. Graph of the inverse Laplace transform.

2.4 Unified Laplace Transform Graph Representation:

By combining the forward and inverse Laplace transforms within a unified graph framework, we observe that each function in the time domain $f(t)$ corresponds to a counterpart in the frequency domain $F(s)$. This results in two vertices connected by two oppositely directed edges, such that each vertex has in-degree 1 and out-degree 1.

In the case of a graph consisting of only two vertices, the resulting structure is a simple 2-cycle (a two-vertex directed cycle). In graphs containing more than one pair of vertices, each pair forms an independent 2-cycle, so the graph can be viewed as a disjoint union of multiple two-vertex cyclic components.

Moreover, the total number of vertices in this model is always even, i.e., $2n$, since applying the Laplace transform to each function generates a corresponding function in the other domain. Therefore, this representation constitutes a new type of graph specific to Laplace transformations, which we propose to call the Laplace Graph (LG).

Theorem1:

Let $V_t = \{f_1(t), f_2(t), f_3(t), \dots, f_n(t)\}$ be the set of time-domain functions and $V_s = \{F_1(s), F_2(s), F_3(s), \dots, F_n(s)\}$ their corresponding Laplace transforms. Then, the Laplace Graph LGLGLG defined on $V = V_t \cup V_s$ satisfies the following properties:

The total number of vertices is always even:

$$|V| = 2n$$

Each pair $(f_i(t), F_i(s))$ forms a 2-cycle (a directed cycle of length two).

The degree of every vertex is equal to 2 (one incoming edge and one outgoing edge).

The graph consists of n disconnected components, each of which is a directed 2-cycle.

Proof:

Forward Laplace transform: For each $f_i(t)$, the forward transform produces $F_i(s) = L\{f_i(t)\}$

This defines a directed edge from $f_i(t)$ to $F_i(s)$.

Inverse Laplace transform: Similarly, the inverse transform gives $f_i(t) = L^{-1}\{F_i(s)\}$

This defines a directed edge from $F_i(s)$ back to $f_i(t)$.

2-cycle formation: Combining the forward and inverse edges, each pair $(f_i(t), F_i(s))$ forms a directed cycle of length 2. Therefore, the degree of each vertex is exactly 2. Multiple components: Applying this process to all n functions produces n independent 2-cycle components. Total vertices: Since each function generates exactly one corresponding transform vertex, the total number of vertices is

$$|V| = 2n$$

which is always even. Since every vertex has exactly one outgoing and one incoming edge, the graph decomposes uniquely into directed cycles, and in this construction, each cycle has length two.

Corollary1: The Laplace Graph LG is isomorphic to the disjoint union of n directed cycles of length 2. Hence, the global structure of the Laplace Graph is determined solely by the number of functions considered.

3. Results and Discussion

From the unified Laplace Transform Graph (LTG), the following properties are observed: No intra-domain edges exist; all edges are cross-domain transformations between the time and frequency domains. In single-domain representations (time-only or frequency-only graphs), each vertex has one outgoing or one incoming transform relation, reflecting the unidirectional nature of the Laplace mapping. In the unified LTG, each vertex has in-degree 1 and out-degree 1, forming a directed 2-cycle between corresponding time-domain and frequency-domain vertices. The LTG consists of multiple weakly connected components, each forming a 2-vertex directed cycle.

4. Conclusion

This paper introduced the Laplace Transform Graph (LTG), a structural graph-theoretic framework that represents functions in both the time and frequency domains within a unified mathematical structure. In this framework, vertices correspond to functions in the respective domains, while directed edges represent the direct and inverse Laplace transform operators.

The proposed model establishes a well-defined bipartite directed graph structure, where each function–transform pair forms a directed 2-cycle. The study derived fundamental structural properties of the LTG, including vertex degree characterization, cycle decomposition, and weak connectivity of components.

These results provide a rigorous foundation for representing transform relations in a graph-theoretic form, enabling both structural analysis and computational interpretation of functional mappings. Furthermore, the LTG framework offers a flexible structure that can be extended to incorporate other integral transforms, such as Fourier and Z-transforms, within a unified graph-based representation.

Overall, the Laplace Transform Graph provides a mathematically consistent bridge between functional analysis and graph theory, offering a structured perspective for both theoretical investigation and computational applications in mathematics and engineering.

References

- [1] [1] G. Doetsch, The Laplace Integral as a Transformation, in: G. Doetsch (Ed.) Introduction to the Theory and Application of the Laplace Transformation, Springer Berlin Heidelberg, Berlin, Heidelberg, 1974, pp. 19-20.
- [2] R.V. Churchill, J.W. Brown, Fourier series and boundary value problems, McGraw-Hill New York 1978.
- [3] S.S.O. SERIES, Schaum's Outline of Theory and Problems of Digital Signal Processing, DOI.
- [4] R. Diestel, Graph minors, Graph Theory, Springer 2017, pp. 347-391.
- [5] D.B. West, Introduction to graph theory, Prentice hall Upper Saddle River 2001.
- [6] F. Harary, Graph theory addison-wesley reading ma usa, HARTIGAN, JA: Clustering Algorithm, DOI (1969).

- [7] A. Farhan Hashosh, H. Basirzadeh, Routh stability criterion and Lyapunov-Routh method in control theory, International Journal of Nonlinear Analysis and Applications, 15 (2024) 111-120.
- [8] A.F. Hashoosh, Special methods controllability and observability in optimal control systems, Al-Kitab Journal for Pure Sciences, 8 (2024) 78-93.
- [9] A.F. Hashoosh, H. Basirzadeh, Stability on optimal control problems and multi-population mathematical models, Util Math, 120 (2023) 500-505.
- [10] D. I. Shuman et al., "The Emerging Field of Signal Processing on Graphs: Extending High-Dimensional Data Analysis to Networks and Other Irregular Domains," IEEE Signal Processing Magazine, 30(3), 83–98, 2013.
- [11] "Graph-theoretic Approach to the Structural Controllability and Observability of Large-scale Systems," 1989.
- [12] Y. Tanaka, "Recent Advances in Signal Processing on Graphs," IEICE ESS Fundamentals Review, 8(1), 15–29, 2014.